

- **Claim: When Metaclass II factorises into a double light cone, the null cone intersection is a line.**

- **Define Lorentz null cones**

$$\text{In[1]:= AAtrans} = \begin{pmatrix} \frac{b1^2}{ww} & 0 & 0 & 0 \\ 0 & -\frac{b1^2}{ww} & 0 & 0 \\ 0 & 0 & -b1 & 0 \\ 0 & 0 & 0 & -b1 \end{pmatrix};$$

$$\text{BBtrans} = \begin{pmatrix} \frac{b1^2(-2+ww)}{ww^2} & \frac{2b1^2}{ww^2} & 0 & 0 \\ \frac{2b1^2}{ww^2} & -\frac{b1^2(2+ww)}{ww^2} & 0 & 0 \\ 0 & 0 & -b1 & 0 \\ 0 & 0 & 0 & -b1 \end{pmatrix};$$

- **Suppose v is a null vector for both Lorentz metrics**

```
In[3]:= v = {a0, a1, a2, a3}
```

```
Out[3]= {a0, a1, a2, a3}
```

```
In[4]:= p0 = v.AAtrans.v;
p1 = v.BBtrans.v;
```

- **If  $p_0 = 0$  and  $p_1 = 0$ , then it follows that  $p_0 - p_1 = 0$ .**

```
In[6]:= Simplify[p0 - p1]
```

$$\text{Out[6]}= \frac{2(a0 - a1)^2 b1^2}{ww^2}$$

- **It follows that  $a_0 = a_1$**

```
In[7]:= Simplify[{p0, p1} /. {a1 -> a0}]
```

$$\text{Out[7]}= \{- (a2^2 + a3^2) b1, - (a2^2 + a3^2) b1\}$$

- **It follows that  $a_2 = a_3 = 0$**

```
In[8]:= Simplify[{p0, p1} /. {a0 -> a1, a2 -> 0, a3 -> 0}]
v /. {a0 -> a1, a2 -> 0, a3 -> 0}
```

$$\text{Out[8]}= \{0, 0\}$$

$$\text{Out[9]}= \{a1, a1, 0, 0\}$$

- **Check**

```
In[10]:= vec = {t, t, 0, 0};
```

```
Simplify[vec.AAtrans.vec]
Simplify[vec.BBtrans.vec]
```

$$\text{Out[11]}= 0$$

$$\text{Out[12]}= 0$$