In[1]:=
```
SetDirectory["~/KappaLib/"];
<< kappaLib-1.2.m
<< helper.m
```

Loading KappaLib v1.2

Loading helper.m..

- **Define Metaclass I**

In[21]:=
$$\texttt{kappa = emMatrixToKappa}\left[\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a2 & 0 & 0 & -b2 & 0 \\ 0 & 0 & a3 & 0 & 0 & a4 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b2 & 0 & 0 & a2 & 0 \\ 0 & 0 & a4 & 0 & 0 & a3 \end{pmatrix}\right];$$

```
sub = {a2 → a1, b2 → b1};
kappa = kappa //. sub;
```

In[7]:=
```
D1 = ( (a1 - a3) ^2 + b1 ^2 - a4 ^2 ) / (b1 a4)
```

Out[7]=
$$\frac{(a1 - a3)^2 - a4^2 + b1^2}{a4\, b1}$$

Case: a1 = a3

- **Define bivector A and metric g**

In[24]:=
$$\texttt{Abivector = }\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix};$$

$$\texttt{Metric = Inverse}\left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & Psi & 0 & 0 \\ 0 & 0 & Psi & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\right];$$

- **Check that metric has Lorentz signature**

In[26]:=
```
Det[Metric]
```

Out[26]=
$$-\frac{1}{Psi^2}$$

- **Formulate equations that should be satisfied**

In[27]:=
```
kappaAlt = C1 emQMedium[SqrtAbsDetG, Inverse[Metric]] +
    emBiProduct[rho, Abivector, Abivector] + C2 emIdentityKappa[];
eqs = Union[Flatten[FullSimplify[kappa - kappaAlt]]];
```

In[29]:=
$$\texttt{sub = }\left\{ Psi \to a4 / b1,\ a3 \to a1,\ C1 \to \frac{b1 / Psi}{SqrtAbsDetG},\ C2 \to a1,\ rho \to \frac{a4^2 + b1^2}{2\, a4} \right\};$$

```
show[simp[eqs //. sub]]
```

Out[30]//MatrixForm=
(1 : 0)

Case: a1 != a3

- **Define A and metric g**

In[32]:=

$$\text{Abivector} = \begin{pmatrix} 0 & 0 & 0 & \frac{-a1+a3}{2\,\text{rho SqrtAbsXi}} \\ 0 & 0 & \text{SqrtAbsXi} & 0 \\ 0 & -\text{SqrtAbsXi} & 0 & 0 \\ -\frac{-a1+a3}{2\,\text{rho SqrtAbsXi}} & 0 & 0 & 0 \end{pmatrix};$$

$$\text{Metric} = \text{Inverse}\left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \text{Psi} & 0 & 0 \\ 0 & 0 & \text{Psi} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\right];$$

- **Show that the equations are satisfied when suitable choice of parameters:**

In[34]:=

```
kappaAlt = C1 emQMedium[SqrtAbsDetG, Inverse[Metric]] +
    emBiProduct[rho, Abivector, Abivector] + C2 emIdentityKappa[];
eqs = Union[Flatten[FullSimplify[kappa - kappaAlt]]];

sub = {
    C2 → a1,
    C1 → b1 (1 / Psi)/SqrtAbsDetG
  };
show[simp[eqs //. sub]]
```

Out[37]//MatrixForm=

$$\begin{pmatrix} 1 & : & 0 \\ 2 & : & a4 - b1\,\text{Psi} - 2\,\text{rho SqrtAbsXi}^2 \\ 3 & : & a4 + \frac{b1}{\text{Psi}} - \frac{(a1-a3)^2}{2\,\text{rho SqrtAbsXi}^2} \end{pmatrix}$$

- **By definition of rho and Xi, these equations are satisfied.**