```
In[2]:= SetDirectory["~/KappaLib/"];
        << kappaLib-1.2.m
        << helper.m

        Loading KappaLib v1.2

        Loading helper.m..
```

In this notebook we define a kappa such that:

1) kappa is algebraically decomposable.

2) beta^2 -alpha gamma = -1.

3) the Fresnel polynomial is product of a two irreducible quadratic forms of signatures + + - -.

4) The nonlinear equation for D has no solution

- **Define medium**

In[5]:= kappa = emMatrixToKappa$\left[\begin{pmatrix} 0 & 0 & -2 & 0 & -1 & 0 \\ -1 & -1 & -1 & \frac{8}{3} & -1 & 0 \\ 1 & 0 & 0 & -\frac{14}{3} & 0 & -1 \\ 0 & 0 & 0 & 0 & 4 & 1 \\ 1 & 0 & -1 & 0 & 2 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 \end{pmatrix}\right]$;

```
In[6]:= (* kappa is invertible *)
        FullSimplify[emDet[kappa]]

        (* kappa has no axion component *)
        Simplify[emTrace[kappa]]

        (* Since this does not simplify to {0}, kappa has a skewon component *)
        Union[Flatten[kappa - emPoincare[kappa]]]

        (* Since this does not simplify to {0}, kappa has a principal component *)
        Union[Flatten[kappa + emPoincare[kappa]]]
```

Out[6]= $-\frac{56}{3}$

Out[7]= 0

Out[8]= $\left\{-5, -\frac{14}{3}, -3, -2, -1, 0, 1, 2, 3, \frac{11}{3}, \frac{14}{3}\right\}$

Out[9]= $\left\{-\frac{14}{3}, -3, -2, -\frac{5}{3}, -1, 0, 1, \frac{5}{3}, 2, 3\right\}$

- **Define constants alpha, beta, gamma and bivectors A and B**

In[10]:=
```
alpha = 2;
beta = 1;
gamma = 1;
rho = 1;
```

$$\text{Abivector} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{7}{2} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{7}{2} & 0 & 0 \end{pmatrix};$$

$$\text{Bbivector} = \begin{pmatrix} 0 & 0 & -\frac{1}{7} & \frac{10}{7} \\ 0 & 0 & \frac{1}{7} & -1 \\ \frac{1}{7} & -\frac{1}{7} & 0 & -1 \\ -\frac{10}{7} & 1 & 1 & 0 \end{pmatrix};$$

- **Note these satisfy beta^2-alpha gamma = 0**

In[16]:=
```
beta ^ 2 - alpha gamma
```

Out[16]= -1

- **Verify that kappa is algebraically decomposable**

In[17]:=
```
LHS = alpha emIdentityKappa[] +
    beta (kappa + emPoincare[kappa]) + gamma emCompose[emPoincare[kappa], kappa];
RHS = emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
Union[Simplify[Flatten[(LHS - RHS)]]]
```

Out[19]= {0}

- **Fresnel polynomial is product of two quadratic forms of signatures (+ + - -)**

In[20]:=
```
coords = {xi0, xi1, xi2, xi3};
fresnel = FullSimplify[emKappaToFresnel[kappa, coords]];
```

In[22]:=

$$\text{gPlus} = \begin{pmatrix} 0 & 3 & 0 & -3 \\ 3 & 6 & 3 & 3 \\ 0 & 3 & 0 & -14 \\ -3 & 3 & -14 & -16 \end{pmatrix};$$

$$\text{gMinus} = \begin{pmatrix} 0 & 3 & -1 & -4 \\ 3 & 6 & 5 & -6 \\ -1 & 5 & 0 & -7 \\ -4 & -6 & -7 & -2 \end{pmatrix};$$

$$\text{frExp} = -\frac{1/4}{3} (\text{coords.gPlus.coords}) (\text{coords.gMinus.coords});$$

In[25]:=
```
Simplify[fresnel - frExp]
```

Out[25]= 0

In[26]:=
```
Sort[Eigenvalues[gPlus] // N]
Sort[Eigenvalues[gMinus] // N]
```

Out[26]= {-25.0281, -0.729749, 6.31277, 9.44509}

Out[27]= {-9.53922, -2.06816, 0.914179, 14.6932}

- **Show that the quadratic factors are irreducible in the complex polynomials**

In[28]:=
```
v = {v0, v1, v2, v3};
w = {w0, w1, w2, w3};
delta = gPlus - (coords.v) (coords.w);
eqs = Union[Flatten[CoefficientList[delta, coords]]];
GroebnerBasis[eqs, Variables[eqs]]

delta = gMinus - (coords.v) (coords.w);
eqs = Union[Flatten[CoefficientList[delta, coords]]];
GroebnerBasis[eqs, Variables[eqs]]
```

Out[32]= {1}

Out[35]= {1}

Show that equation

D (kappa + beta Id) = 1/2 trace (rho bar(D) otimes D) A + B

has no (complex) solution for D

In[36]:=
```
(*
 * If D=1/2 D^ij d/dx^i /\ d/dx^j is a bivector we represent
 * the coefficients by the anti-symmetric matrix with coefficients
 * of (D^ij)_ij. If kappa is an antisymmetric (2,2)-tensor,
 * then this routine returns coefficients of bivector D (kappa).
 *)
contract[biv_,kappa_] := Table[
 1/2 Sum[biv[[i]][[j]]emReadNormal[kappa,a,b,i,j]
       ,
       {i, 1, 4},{j, 1, 4}
     ]
 ,
 {a, 1, 4}, {b, 1, 4}
]
```

In[37]:=
```
Dbivector = emMatrix["d", 4, Structure → "AntiSymmetric"];
dLHS = contract[Dbivector, kappa + beta emIdentityKappa[]];
dRHS = 1 / 2 emTrace[emBiProduct[rho, Dbivector, Dbivector]] Abivector + Bbivector;
deqs = Union[Flatten[Simplify[dLHS - dRHS]]];
GroebnerBasis[deqs, Variables[deqs]]
```

Out[41]= {1}