```
In[2]:= SetDirectory["~/KappaLib/"];
        << kappaLib-1.2.m
        << helper.m

        Loading KappaLib v1.2

        Loading helper.m..
```

- **In this notebook we define a 5-parameter class of mediums:**

  1) each kappa is algebraically decomposable

  2) beta^2 -alpha gamma = 0

  3) the Fresnel polynomial always has a linear factor

- **Define medium**

In[5]:= kappa = emMatrixToKappa$\left[\begin{pmatrix} 1 & 1 & 0 & 1 & -1 & k16 \\ k21 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ k41 & 0 & 0 & -1 & 1 & 1 \\ k41 & 0 & 0 & 0 & 0 & 1 \\ k41 + k62 - k16\,k41\,k62 + \frac{1}{2}(-1 + k21\,k64) & k62 & 1 & k64 & 0 & 1 \end{pmatrix}\right]$;

```
In[6]:= (* kappa is invertible *)
        FullSimplify[emDet[kappa]]

        (* kappa has no axion component *)
        Simplify[emTrace[kappa]]

        (* Since this does not simplify to {0}, kappa has a skewon component *)
        Union[Flatten[kappa - emPoincare[kappa]]]

        (* Since this does not simplify to {0}, kappa has a principal component *)
        Union[Flatten[kappa + emPoincare[kappa]]]
```

Out[6]= 1

Out[7]= 0

Out[8]= $\{-2, -1, 0, 1, 2, -k16, k16, -1 + k21, -k41, k62, -k64, k64, \frac{1}{4}(2 - 4\,k41 - 4\,k62 + 4\,k16\,k41\,k62 - 2\,k21\,k64), k41 + k62 - k16\,k41\,k62 + \frac{1}{2}(-1 + k21\,k64)\}$

Out[9]= $\{-2, -1, 0, 1, 2, k16, -1 - k21, 1 + k21, -k41, k41, 2\,k41, -k62, k62, k64, \frac{1}{4}(-2 + 4\,k41 + 4\,k62 - 4\,k16\,k41\,k62 + 2\,k21\,k64), k41 + k62 - k16\,k41\,k62 + \frac{1}{2}(-1 + k21\,k64)\}$

- **Kappa depends on 5-variables**

```
In[10]:= Variables[kappa]
```

Out[10]= {k16, k21, k41, k62, k64}

- **Define constants alpha, beta, gamma and bivectors A and B:**

In[11]:=
```
alpha = 1;
beta = 1;
gamma = 1;
rho = 1;
```

$$\text{Abivector} = \begin{pmatrix} 0 & k41 & 0 & 0 \\ -k41 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix};$$

$$\text{Bbivector} = \begin{pmatrix} 0 & \frac{2+k21}{2} & 1 & 0 \\ -1-\frac{k21}{2} & 0 & \frac{k16}{2} & 0 \\ -1 & -\frac{k16}{2} & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix};$$

- **Note these satisfy beta^2-alpha gamma = 0**

In[17]:=
```
beta ^ 2 - alpha gamma
```

Out[17]= 0

- **Verify that is algebraically decomposable**

In[18]:=
```
LHS  = alpha emIdentityKappa[] +
    beta (kappa + emPoincare[kappa]) + gamma emCompose[emPoincare[kappa], kappa];
RHS = emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
Union[Simplify[Flatten[(LHS - RHS)]]]
```

Out[20]= {0}

- **Compute Fresnel polynomial**

In[21]:=
```
coords = {xi0, xi1, xi2, xi3};
fresnel = FullSimplify[emKappaToFresnel[kappa, coords]];
```

- **Fresnel polynomial is product of linear factor and 3rd order factor**

In[23]:=

$$\text{factor1} = \frac{1}{2}\,\text{xi3};$$

$$\begin{aligned}\text{factor2} = \Big(&2\,\text{xi1}\,(\text{xi1}+\text{xi2})\,((2+\text{k21}+\text{k16}\,\text{k21})\,\text{xi1}+(\text{k16}+\text{k21})\,\text{xi2})\,+\\ &\Big((12+\text{k16}+2\,\text{k21}-2\,\text{k16}\,\text{k62}-\text{k16}\,\text{k21}\,\text{k64})\,\text{xi1}^2-\\ &\quad(-4+\text{k16}-2\,\text{k21}+\text{k16}\,\text{k21}\,\text{k64})\,\text{xi1}\,\text{xi2}+2\,\text{k16}\,(\text{k62}-\text{k64})\,\text{xi2}^2\Big)\,\text{xi3}\,-\\ &2\,((1+\text{k16}-2\,\text{k62}+2\,\text{k64})\,\text{xi1}-(\text{k16}+2\,\text{k62}-2\,\text{k64})\,\text{xi2})\,\text{xi3}^2+4\,\text{xi3}^3+\\ &2\,\text{xi0}^2\,((3+\text{k21}+\text{k21}\,\text{k62}-\text{k21}\,\text{k64})\,\text{xi1}+2\,\text{xi2}-\text{k21}\,\text{xi3})\,+\\ &2\,\text{k41}\,\Big(-2\,\text{xi0}^3+\text{xi0}\,\text{xi1}\,((-2+\text{k16}\,(-1+\text{k16}\,\text{k62}))\,\text{xi1}+\text{k16}\,(2\,\text{k62}-\text{k64})\,\text{xi2})\,+\\ &\quad\text{xi0}^2\,((-2-\text{k16}+2\,\text{k16}\,\text{k62})\,\text{xi1}+2\,(\text{k62}-\text{k64})\,\text{xi2}+2\,\text{xi3})\,-\\ &\quad\text{xi1}\,(\text{k16}\,\text{xi1}-\text{k16}\,\text{xi2}-2\,\text{xi3})\,(\text{xi1}+\text{xi2}+\text{xi3}-\text{k16}\,\text{k62}\,\text{xi3})\Big)\,+\\ &\text{xi0}\,\Big((4+\text{k16}+6\,\text{k21}-2\,\text{k16}\,\text{k62}-\text{k16}\,\text{k21}\,\text{k64})\,\text{xi1}^2+4\,(-\text{k62}+\text{k64})\,\text{xi2}^2-\\ &\quad2\,(2+\text{k16}-\text{k21}\,\text{k62}+\text{k21}\,\text{k64})\,\text{xi2}\,\text{xi3}+2\,(-2+\text{k21})\,\text{xi3}^2+2\,\text{xi1}\\ &\quad\;((3+\text{k21}-(2+\text{k16})\,\text{k62}+2\,\text{k64})\,\text{xi2}-(\text{k16}\,(1+\text{k21})-(2+\text{k21})\,(\text{k62}-\text{k64}))\,\text{xi3})\Big)\Big);\end{aligned}$$

```
Simplify[fresnel - factor1 factor2]
```

Out[25]= 0

Show that

D (kappa + beta Id) = 1/2 trace (rho bar(D) otimes D) A + B

has a solution for D

In[26]:=
```
(*
 * If D=1/2 D^ij d/dx^i /\ d/dx^j is a bivector we represent
 * the coefficients by the anti-symmetric matrix with coefficients
 * of (D^ij)_ij. If kappa is an antisymmetric (2,2)-tensor,
 * then this routine returns coefficients of bivector D (kappa).
 *)
contract[biv_,kappa_] := Table[
 1/2 Sum[biv[[i]][[j]]emReadNormal[kappa,a,b,i,j]
      ,
      {i, 1, 4},{j, 1, 4}
    ]
 ,
 {a, 1, 4}, {b, 1, 4}
]
```

In[27]:= **Dbivector** = $\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$;

```
dLHS = contract[Dbivector, kappa + beta emIdentityKappa[]];
dRHS = 1 / 2 emTrace[emBiProduct[rho, Dbivector, Dbivector]] Abivector + Bbivector;
deqs = Union[Flatten[Simplify[dLHS - dRHS]]]
```

Out[30]= {0}

In[31]:= `1 / 2 emTrace[emBiProduct[rho, Dbivector, Dbivector]]`

Out[31]= 0