

```

In[1]:= SetDirectory["~/KappaLib"];
<< kappaLib-1.2.m
<< helper.m

Loading KappaLib v1.2

Loading helper.m..

```

- Define Metaclass VII with parameters:

α_i in \mathbb{R} , β_i in $\mathbb{R} \setminus 0$, and β_i all have same sign.

```

In[4]:= kappa = emMatrixToKappa [
  (
    a1  0  0  a4  0  0
    0  a2  0  0  a5  0
    0  0  a3  0  0  a6
    a4  0  0  a1  0  0
    0  a5  0  0  a2  0
    0  0  a6  0  0  a3
  )
];

```

Write out algebraic equations that kappa satisfies and eliminate variables for A and B

```

In[5]:= eta = kappa + mu emIdentityKappa[];
LHS = emCompose[eta, eta];
AA = emMatrix["A", 4, Structure -> "AntiSymmetric"];
BB = emMatrix["B", 4, Structure -> "AntiSymmetric"];
RHS = -lambda emIdentityKappa[] + emBiProduct[rho, AA, BB] + emBiProduct[rho, BB, AA];

```

- Since rho, A,B are all non-zero, we may scale A and assume that rho = 1

```

In[10]:= rho = 1;

```


In[18]:= show[gb]

Out[18]/MatrixForm=

$$\begin{array}{l}
1 : \quad a_4 a_5 a_6 (a_1 + \mu) (a_2 + \mu) (a_3 + \mu) \\
2 : \quad a_4 a_5 a_6 (a_6^2 + \lambda) (a_1 + \mu) (a_2 + \mu) \\
3 : \quad a_4 a_5 a_6 (a_5^2 + \lambda) (a_1 + \mu) (a_3 + \mu) \\
4 : \quad a_4 a_5 a_6 (a_4^2 + \lambda) (a_2 + \mu) (a_3 + \mu) \\
5 : \quad a_4 a_5 a_6 (a_5^2 + \lambda) (a_6^2 + \lambda) (a_1 + \mu) \\
6 : \quad a_4 a_5 a_6 (a_4^2 + \lambda) (a_6^2 + \lambda) (a_2 + \mu) \\
7 : \quad a_4 a_5 a_6 (a_4^2 + \lambda) (a_5^2 + \lambda) (a_3 + \mu) \\
8 : \quad a_5 a_6 (a_2 + \mu) (a_3 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2) \\
9 : \quad a_4 a_6 (a_1 + \mu) (a_3 + \mu) (a_5^2 + \lambda + (a_2 + \mu)^2) \\
10 : \quad a_4 a_5 (a_1 + \mu) (a_2 + \mu) (a_6^2 + \lambda + (a_3 + \mu)^2) \\
11 : \quad a_4 a_5 a_6 (a_4^2 + \lambda) (a_5^2 + \lambda) (a_6^2 + \lambda) \\
12 : \quad a_5 a_6 (a_6^2 + \lambda) (a_2 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2) \\
13 : \quad a_5 a_6 (a_5^2 + \lambda) (a_3 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2) \\
14 : \quad a_4 a_6 (a_6^2 + \lambda) (a_1 + \mu) (a_5^2 + \lambda + (a_2 + \mu)^2) \\
15 : \quad a_4 a_6 (a_4^2 + \lambda) (a_3 + \mu) (a_5^2 + \lambda + (a_2 + \mu)^2) \\
16 : \quad a_4 a_5 (a_5^2 + \lambda) (a_1 + \mu) (a_6^2 + \lambda + (a_3 + \mu)^2) \\
17 : \quad a_4 a_5 (a_4^2 + \lambda) (a_2 + \mu) (a_6^2 + \lambda + (a_3 + \mu)^2) \\
18 : \quad a_5 a_6 (a_5^2 + \lambda) (a_6^2 + \lambda) (a_4^2 + \lambda + (a_1 + \mu)^2) \\
19 : \quad a_4 a_6 (a_4^2 + \lambda) (a_6^2 + \lambda) (a_5^2 + \lambda + (a_2 + \mu)^2) \\
20 : \quad a_4 a_5 (a_4^2 + \lambda) (a_5^2 + \lambda) (a_6^2 + \lambda + (a_3 + \mu)^2) \\
21 : \quad a_5 (a_2 + \mu) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2) \\
22 : \quad a_6 (a_3 + \mu) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2) \\
23 : \quad a_4 (a_1 + \mu) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2) \\
24 : \quad a_6 (a_3 + \mu) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2) \\
25 : \quad a_4 (a_1 + \mu) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2) \\
26 : \quad a_5 (a_2 + \mu) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2) \\
27 : \quad a_6 (a_3 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2) (a_5^2 + \lambda + (a_2 + \mu)^2) \\
28 : \quad a_5 (a_2 + \mu) (a_4^2 + \lambda + (a_1 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2) \\
29 : \quad a_4 (a_1 + \mu) (a_5^2 + \lambda + (a_2 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2) \\
30 : \quad a_5 (a_5^2 + \lambda) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2) \\
31 : \quad a_6 (a_6^2 + \lambda) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2) \\
32 : \quad a_4 (a_4^2 + \lambda) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2) \\
33 : \quad a_6 (a_6^2 + \lambda) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2) \\
34 : \quad a_4 (a_4^2 + \lambda) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2) \\
35 : \quad a_5 (a_5^2 + \lambda) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2) \\
36 : \quad a_6 (a_6^2 + \lambda) (a_4^2 + \lambda + (a_1 + \mu)^2) (a_5^2 + \lambda + (a_2 + \mu)^2) \\
37 : \quad a_5 (a_5^2 + \lambda) (a_4^2 + \lambda + (a_1 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2) \\
38 : \quad a_4 (a_4^2 + \lambda) (a_5^2 + \lambda + (a_2 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2) \\
39 : \quad (a_5^2 + \lambda + (a_2 + \mu)^2) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2) \\
40 : \quad (a_6^2 + \lambda + (a_3 + \mu)^2) (\lambda + (a_1 - a_4 + \mu)^2) (\lambda + (a_1 + a_4 + \mu)^2) \\
41 : \quad (a_4^2 + \lambda + (a_1 + \mu)^2) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2) \\
42 : \quad (a_6^2 + \lambda + (a_3 + \mu)^2) (\lambda + (a_2 - a_5 + \mu)^2) (\lambda + (a_2 + a_5 + \mu)^2) \\
43 : \quad (a_4^2 + \lambda + (a_1 + \mu)^2) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2) \\
44 : \quad (a_5^2 + \lambda + (a_2 + \mu)^2) (\lambda + (a_3 - a_6 + \mu)^2) (\lambda + (a_3 + a_6 + \mu)^2) \\
45 : \quad (a_4^2 + \lambda + (a_1 + \mu)^2) (a_5^2 + \lambda + (a_2 + \mu)^2) (a_6^2 + \lambda + (a_3 + \mu)^2)
\end{array}$$

- Equation $gb[[45]]=0$ contradicts $\lambda > 0$.

In[19]:= `show[{gb[[45]]}]`

Out[19]/MatrixForm=

$$\left(1 \quad : \quad (a_4^2 + \lambda + (a_1 + \mu)^2) \quad (a_5^2 + \lambda + (a_2 + \mu)^2) \quad (a_6^2 + \lambda + (a_3 + \mu)^2) \right)$$

- Thus κ can not be in Metaclass VII