In[1]:= **SetDirectory["~/KappaLib"];**
<< kappaLib-1.2.m
<< helper.m

Loading KappaLib v1.2

Loading helper.m..

- **Define Metaclass VI with parameters:**

    alpha_i in R,   beta_i in R\0,   and   beta_i all have same sign.

In[4]:= **kappa = emMatrixToKappa** $\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a2 & 0 & 0 & a4 & 0 \\ 0 & 0 & a3 & 0 & 0 & a5 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & a4 & 0 & 0 & a2 & 0 \\ 0 & 0 & a5 & 0 & 0 & a3 \end{pmatrix}$ **];**

Write out algebraic equations that kappa satisfies and eliminate variables for A and B

In[5]:= **eta = kappa + mu emIdentityKappa[];**
LHS = emCompose[eta, eta];
AA = emMatrix["A", 4, Structure → "AntiSymmetric"];
BB = emMatrix["B", 4, Structure → "AntiSymmetric"];
RHS = -lambda emIdentityKappa[] + emBiProduct[rho, AA, BB] + emBiProduct[rho, BB, AA];

- **Since rho, A,B are all non-zero, we may scale A and assume that rho = 1**

In[10]:= **rho = 1;**

```
In[11]:= eqs = simp[Union[Flatten[LHS - RHS]]];
        show[eqs]
```

Out[12]//MatrixForm=

$$
\begin{pmatrix}
1 & : & 0 \\
2 & : & 2\,(A13\,B12 + A12\,B13) \\
3 & : & 2\,(A14\,B13 + A13\,B14) \\
4 & : & 2\,(A23\,B13 + A13\,B23) \\
5 & : & 2\,(A24\,B12 + A12\,B24) \\
6 & : & 2\,(A24\,B14 + A14\,B24) \\
7 & : & 2\,(A24\,B23 + A23\,B24) \\
8 & : & 2\,(A34\,B13 + A13\,B34) \\
9 & : & 2\,(A34\,B24 + A24\,B34) \\
10 & : & -2\,(A13\,B12 + A12\,B13) \\
11 & : & -2\,(A14\,B12 + A12\,B14) \\
12 & : & -2\,(A14\,B13 + A13\,B14) \\
13 & : & -2\,(A23\,B12 + A12\,B23) \\
14 & : & -2\,(A23\,B13 + A13\,B23) \\
15 & : & -2\,(A24\,B12 + A12\,B24) \\
16 & : & -2\,(A24\,B14 + A14\,B24) \\
17 & : & -2\,(A24\,B23 + A23\,B24) \\
18 & : & -2\,(A34\,B13 + A13\,B34) \\
19 & : & -2\,(A34\,B14 + A14\,B34) \\
20 & : & -2\,(A34\,B23 + A23\,B34) \\
21 & : & -2\,(A34\,B24 + A24\,B34) \\
22 & : & 4\,A13\,B13 - 2\,a4\,(a2 + mu) \\
23 & : & 4\,A24\,B24 - 2\,a4\,(a2 + mu) \\
24 & : & -4\,A34\,B34 - 2\,b1\,(a1 + mu) \\
25 & : & -4\,A12\,B12 + 2\,b1\,(a1 + mu) \\
26 & : & -4\,A14\,B14 + 2\,a5\,(a3 + mu) \\
27 & : & -4\,A23\,B23 + 2\,a5\,(a3 + mu) \\
28 & : & a4^2 + 2\,A24\,B13 + 2\,A13\,B24 + lambda + (a2 + mu)^2 \\
29 & : & a5^2 - 2\,A23\,B14 - 2\,A14\,B23 + lambda + (a3 + mu)^2 \\
30 & : & -b1^2 - 2\,A34\,B12 - 2\,A12\,B34 + lambda + (a1 + mu)^2
\end{pmatrix}
$$

```
In[13]:= elimVars = Join[Variables[AA], Variables[BB]]
```

Out[13]= {A12, A13, A14, A23, A24, A34, B12, B13, B14, B23, B24, B34}

```
In[14]:= condVars = Join[Variables[kappa], {lambda, mu}]
```

Out[14]= {a1, a2, a3, a4, a5, b1, lambda, mu}

■ Eliminate variables using a Gröbner basis

```
In[15]:= gb = GroebnerBasis[eqs, condVars, elimVars]; // Timing
        gb = simp[gb]; // Timing
        Length[gb]
```

Out[15]= {131.666, Null}

Out[16]= {1.17233, Null}

Out[17]= 45

In[18]:= **show[gb]**

Out[18]//MatrixForm=

$$1 \;:\; a4\;a5\;b1\;(a1+mu)\;(a2+mu)\;(a3+mu)$$

$$2 \;:\; a4\;a5\;b1\;\left(a5^2+lambda\right)\;(a1+mu)\;(a2+mu$$

$$3 \;:\; a4\;a5\;b1\;\left(a4^2+lambda\right)\;(a1+mu)\;(a3+mu$$

$$4 \;:\; a4\;a5\;b1\;\left(b1^2-lambda\right)\;(a2+mu)\;(a3+mu$$

$$5 \;:\; a4\;a5\;b1\;\left(a4^2+lambda\right)\;\left(a5^2+lambda\right)\;(a1$$

$$6 \;:\; a4\;a5\;b1\;\left(b1^2-lambda\right)\;\left(a5^2+lambda\right)\;(a2$$

$$7 \;:\; a4\;a5\;b1\;\left(b1^2-lambda\right)\;\left(a4^2+lambda\right)\;(a3$$

$$8 \;:\; a5\;b1\;(a1+mu)\;(a3+mu)\;\left(a4^2+lambda+(a2+\right.$$

$$9 \;:\; a4\;b1\;(a1+mu)\;(a2+mu)\;\left(a5^2+lambda+(a3+\right.$$

$$10 \;:\; a4\;a5\;b1\;\left(b1^2-lambda\right)\;\left(a4^2+lambda\right)\;\left(a5^2+\right.$$

$$11 \;:\; a4\;a5\;(a2+mu)\;(a3+mu)\;\left(-b1^2+lambda+(a1\right.$$

$$12 \;:\; a5\;b1\;\left(a5^2+lambda\right)\;(a1+mu)\;\left(a4^2+lambda+(a\right.$$

$$13 \;:\; a5\;b1\;\left(b1^2-lambda\right)\;(a3+mu)\;\left(a4^2+lambda+(a\right.$$

$$14 \;:\; a4\;b1\;\left(a4^2+lambda\right)\;(a1+mu)\;\left(a5^2+lambda+(a\right.$$

$$15 \;:\; a4\;b1\;\left(b1^2-lambda\right)\;(a2+mu)\;\left(a5^2+lambda+(a\right.$$

$$16 \;:\; a4\;a5\;\left(a5^2+lambda\right)\;(a2+mu)\;\left(-b1^2+lambda+(\right.$$

$$17 \;:\; a4\;a5\;\left(a4^2+lambda\right)\;(a3+mu)\;\left(-b1^2+lambda+(\right.$$

$$18 \;:\; a5\;b1\;\left(b1^2-lambda\right)\;\left(a5^2+lambda\right)\;\left(a4^2+lambda+\right.$$

$$19 \;:\; a4\;b1\;\left(b1^2-lambda\right)\;\left(a4^2+lambda\right)\;\left(a5^2+lambda+\right.$$

$$20 \;:\; a4\;a5\;\left(a4^2+lambda\right)\;\left(a5^2+lambda\right)\;\left(-b1^2+lambda\right.$$

$$21 \;:\; b1\;(a1+mu)\;\left(lambda+(a2-a4+mu)^2\right)\;\left(lambda+(a\right.$$

$$22 \;:\; a5\;(a3+mu)\;\left(lambda+(a2-a4+mu)^2\right)\;\left(lambda+(a\right.$$

$$23 \;:\; b1\;(a1+mu)\;\left(lambda+(a3-a5+mu)^2\right)\;\left(lambda+(a\right.$$

$$24 \;:\; a4\;(a2+mu)\;\left(lambda+(a3-a5+mu)^2\right)\;\left(lambda+(a\right.$$

$$25 \;:\; b1\;(a1+mu)\;\left(a4^2+lambda+(a2+mu)^2\right)\;\left(a5^2+lambda\right.$$

$$26 \;:\; a5\;(a3+mu)\;\left(-b1^2+lambda+(a1+mu)^2\right)\;\left(a4^2+lambd\right.$$

$$27 \;:\; a4\;(a2+mu)\;\left(-b1^2+lambda+(a1+mu)^2\right)\;\left(a5^2+lambd\right.$$

$$28 \;:\; b1\;\left(b1^2-lambda\right)\;\left(lambda+(a2-a4+mu)^2\right)\;\left(lambda+\right.$$

$$29 \;:\; a5\;\left(a5^2+lambda\right)\;\left(lambda+(a2-a4+mu)^2\right)\;\left(lambda+\right.$$

$$30 \;:\; b1\;\left(b1^2-lambda\right)\;\left(lambda+(a3-a5+mu)^2\right)\;\left(lambda+\right.$$

$$31 \;:\; a4\;\left(a4^2+lambda\right)\;\left(lambda+(a3-a5+mu)^2\right)\;\left(lambda+\right.$$

$$32 \;:\; b1\;\left(b1^2-lambda\right)\;\left(a4^2+lambda+(a2+mu)^2\right)\;\left(a5^2+lam\right.$$

$$33 \;:\; a5\;\left(a5^2+lambda\right)\;\left(-b1^2+lambda+(a1+mu)^2\right)\;\left(a4^2+lar\right.$$

$$34 \;:\; a4\;\left(a4^2+lambda\right)\;\left(-b1^2+lambda+(a1+mu)^2\right)\;\left(a5^2+lar\right.$$

$$35 \;:\; \left(a5^2+lambda+(a3+mu)^2\right)\;\left(lambda+(a2-a4+mu)^2\right)\;\left(lamb\right.$$

$$36 \;:\; \left(a4^2+lambda+(a2+mu)^2\right)\;\left(lambda+(a3-a5+mu)^2\right)\;\left(lamb\right.$$

$$37 \;:\; \left(-b1^2+lambda+(a1+mu)^2\right)\;\left(lambda+(a2-a4+mu)^2\right)\;\left(lam\right.$$

$$38 \;:\; \left(-b1^2+lambda+(a1+mu)^2\right)\;\left(lambda+(a3-a5+mu)^2\right)\;\left(lam\right.$$

$$39 \;:\; \left(-b1^2+lambda+(a1+mu)^2\right)\;\left(a4^2+lambda+(a2+mu)^2\right)\;\left(a5^2\right.$$

$$40 \;:\; a4\;(a2+mu)\;\left(a1^4+b1^4+4\;a1^3\;mu+2\;b1^2\left(-lambda+mu^2\right)+\left(lambda+mu^2\right)^2+4\;a1\;mu\;(b1\right.$$

$$41 \;:\; a5\;(a3+mu)\;\left(a1^4+b1^4+4\;a1^3\;mu+2\;b1^2\left(-lambda+mu^2\right)+\left(lambda+mu^2\right)^2+4\;a1\;mu\;(b1\right.$$

$$42 \;:\; a4\;\left(a4^2+lambda\right)\;\left(a1^4+b1^4+4\;a1^3\;mu+2\;b1^2\left(-lambda+mu^2\right)+\left(lambda+mu^2\right)^2+4\;a1\;mu\right.$$

$$43 \;:\; a5\;\left(a5^2+lambda\right)\;\left(a1^4+b1^4+4\;a1^3\;mu+2\;b1^2\left(-lambda+mu^2\right)+\left(lambda+mu^2\right)^2+4\;a1\;mu\right.$$

$$44 \;:\; \left(a4^2+lambda+(a2+mu)^2\right)\;\left(a1^4+b1^4+4\;a1^3\;mu+2\;b1^2\left(-lambda+mu^2\right)+\left(lambda+mu^2\right)^2+4\;a\right.$$

$$45 \;:\; \left(a5^2+lambda+(a3+mu)^2\right)\;\left(a1^4+b1^4+4\;a1^3\;mu+2\;b1^2\left(-lambda+mu^2\right)+\left(lambda+mu^2\right)^2+4\;a\right.$$

- **Equation gb[[35]]=0 contradicts lambda > 0.**

In[19]:= **show[{gb[[35]]}]**

Out[19]//MatrixForm=

$$\left(\begin{array}{ccc} 1 & : & \left(a5^2 + \text{lambda} + (a3 + \text{mu})^2 \right) \left(\text{lambda} + (a2 - a4 + \text{mu})^2 \right) \left(\text{lambda} + (a2 + a4 + \text{mu})^2 \right) \end{array} \right)$$

- **Thus kappa can not be in Metaclass VI**