In[1]:=
```
SetDirectory["~/KappaLib"];
<< kappaLib-1.2.m
```

```
Loading KappaLib v1.2
```

- **Metaclass I:**

In[3]:=
$$\texttt{kappa = emMatrixToKappa}\left[\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a2 & 0 & 0 & -b2 & 0 \\ 0 & 0 & a3 & 0 & 0 & -b3 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b2 & 0 & 0 & a2 & 0 \\ 0 & 0 & b3 & 0 & 0 & a3 \end{pmatrix}\right];$$

- **By Theorem 3.5 we may assume that a2 = a3 and b2 = b3**

In[4]:=
```
sub = {a3 → a2, b3 → b2}
```

Out[4]=
```
{a3 → a2, b3 → b2}
```

- **In addition, let us assume that a1 = a2**

In[5]:=
```
sub = Append[sub, a2 → a1]
```

Out[5]=
```
{a3 → a2, b3 → b2, a2 → a1}
```

In[6]:=
```
kappa = kappa //. sub;
```

- **Medium characteristics**

In[7]:=
```
emKappaToMatrix[kappa] // MatrixForm
FullSimplify[emDet[kappa]]
Simplify[emTrace[kappa]]
```

Out[7]//MatrixForm=

$$\begin{pmatrix} a1 & 0 & 0 & -b1 & 0 & 0 \\ 0 & a1 & 0 & 0 & -b2 & 0 \\ 0 & 0 & a1 & 0 & 0 & -b2 \\ b1 & 0 & 0 & a1 & 0 & 0 \\ 0 & b2 & 0 & 0 & a1 & 0 \\ 0 & 0 & b2 & 0 & 0 & a1 \end{pmatrix}$$

Out[8]= $\left(a1^2 + b1^2\right) \left(a1^2 + b2^2\right)^2$

Out[9]= 6 a1

- **Define coefficients in Theorem 5.1:**
- **Note: Since a1=a2, we may assume that b1 != b2 (by representation theorem of medium with double light cone)**

In[10]:=
```
rho = (b2^2 - b1^2) / 2;
mu = -a1;
lambda = b2^2;
```

$$\texttt{Abivector = }\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

$$\texttt{Bbivector = }\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix};$$

- **Verify claim:**

In[15]:=
```
eta = kappa + mu emIdentityKappa[];
LHS = emCompose[eta, eta];
RHS = -lambda emIdentityKappa[] +
   emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
Union[Flatten[LHS - RHS]]
```

Out[18]= {0}

- **Note:**
  *** kappa + mu Id is trace-free in this case**
  *** lambda > 0**
  *** rho, A, B are all non-zero**

In[19]:=
```
emTrace[eta]
```

Out[19]= 0

Solvability of equation for D

- **Define constants alpha, beta, gamma that appear in definition of algebraically decomposable medium**

In[20]:=
```
alpha = lambda + mu^2;
beta = mu;
gamma = 1;
```

- **Explicitly verify that kappa is algebraically decomposable:**

In[23]:=
```
LHS = alpha emIdentityKappa[] +
   beta (kappa + emPoincare[kappa]) + gamma emCompose[emPoincare[kappa], kappa];
RHS = emBiProduct[rho, Abivector, Bbivector] + emBiProduct[rho, Bbivector, Abivector];
eqs = Union[Simplify[Flatten[(LHS - RHS)]]]
```

Out[25]= {0}

- **Existence of D:**

In[26]:=
```
Dbivector = -1 / b1 Abivector;

(* contract kappa with a bivector from the left *)
contract[biv_, kappa_] := Table[
  1 / 2 Sum[
    biv[[i]][[j]] emReadNormal[kappa, a, b, i, j]
    ,
    {i, 1, 4}, {j, 1, 4}
   ]
  ,
  {a, 1, 4}, {b, 1, 4}
 ]
dLHS = contract[Dbivector, kappa + beta emIdentityKappa[]];
dRHS = 1 / 2 emTrace[emBiProduct[rho, Dbivector, Dbivector]] Abivector + Bbivector;
deqs = Union[Flatten[Simplify[dLHS - dRHS]]]
```

Out[30]= {0}