

- To list possible S matrices for a metaclass, run the below code once B,W and V matrices are defined for that metaclass.
- The below code works by permutating eigenvectors for eigenvalues +1 and -1 in W.

```

W = W /. {eps1 → -1, eps2 → -1, eps3 → -1};
For[i = 1, i ≤ 6, i++,

  For[j = 1, j ≤ 6, j++,

    tmp = Eigenvectors[W];
    kk1 = Permutations[{1, 2, 3}][[i]];
    kk2 = Permutations[{4, 5, 6}][[j]];

    tmpP = {tmp[[kk1[[1]]]], tmp[[kk1[[2]]]],
            tmp[[kk1[[3]]]], tmp[[kk2[[1]]]], tmp[[kk2[[2]]]], tmp[[kk2[[3]]]]};
    dd = tmpP.W.Transpose[tmpP];
    scale = DiagonalMatrix[Table[Sqrt[Abs[dd[[i]][[i]]]], {i, 1, 6}]];
    scale = Inverse[scale];
    S = Sqrt[2] Inverse[Eigenvectors[B]].scale.tmpP;
    res = Simplify[S.V.Inverse[S]];

    If[Count[Flatten[res], 0] ≥ 10,
      Print[" "];
      Print[" "];
      Print["Permutation: " <> ToString[i] <> ToString[j]];
      Print["Zeroes: " <> ToString[Count[Flatten[res], 0]]];
      Print[S // MatrixForm];
      Print[Transpose[S].B.S == W];
      (*Print[res//MatrixForm];*)
      Print[Petrov[res]];
    ];
  ]
]

```

Permutation: 11

Zeroes: 16

$$\begin{pmatrix}
-\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\
-\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
-\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}
\end{pmatrix}$$

True

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \text{lam1} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 & 0 & \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{2\sqrt{2}} \\ 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 \\ \frac{1}{2\sqrt{2}} & \frac{\text{lam1} + \text{lam3}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) & -\frac{1}{2\sqrt{2}} \\ \text{lam1} & \frac{1}{2\sqrt{2}} & 0 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Permutation: 12

Zeros: 22

$$\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam1} + \text{lam3}) & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{\text{lam1} + \text{lam3}}{2} \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 \\ 0 & \text{lam1} & 0 & 0 & 0 & 0 \\ \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) \end{pmatrix}$$

Permutation: 13

Zeros: 16

$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \text{lam1} \\ -\frac{1}{2\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 & 0 & \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{2\sqrt{2}} \\ 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 \\ \frac{1}{2\sqrt{2}} & \frac{\text{lam1} + \text{lam4}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) & -\frac{1}{2\sqrt{2}} \\ \text{lam1} & \frac{1}{2\sqrt{2}} & 0 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Permutation: 14

Zeroes: 22

$$\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam1} + \text{lam4}) & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{\text{lam1} + \text{lam4}}{2} \\ -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 \\ 0 & \text{lam1} & 0 & 0 & 0 & 0 \\ \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) \end{pmatrix}$$

Permutation: 21

Zeroes: 22

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

True

$$\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & \text{lam1} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 & 0 & \frac{\text{lam1} + \text{lam3}}{2} & 0 \\ 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{\text{lam1} + \text{lam3}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 \\ \text{lam1} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Permutation: 22

Zeroes: 16

$$\begin{pmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam1} + \text{lam3}) & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{\text{lam1} + \text{lam3}}{2} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & \text{lam1} & \frac{1}{2\sqrt{2}} \\ 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 \\ \frac{1}{2\sqrt{2}} & \text{lam1} & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{2\sqrt{2}} & 0 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam3}) \end{pmatrix}$$

Permutation: 23

Zeroes: 22

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & \text{lam1} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 & 0 & \frac{\text{lam1} + \text{lam4}}{2} & 0 \\ 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{\text{lam1} + \text{lam4}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 \\ \text{lam1} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Permutation: 24

Zeroes: 16

$$\begin{pmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam1} + \text{lam4}) & -\frac{1}{2\sqrt{2}} & 0 & 0 & \frac{1}{2\sqrt{2}} & \frac{\text{lam1} + \text{lam4}}{2} \\ -\frac{1}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & \text{lam1} & \frac{1}{2\sqrt{2}} \\ 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 \\ \frac{1}{2\sqrt{2}} & \text{lam1} & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} \\ \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{2\sqrt{2}} & 0 & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam4}) \end{pmatrix}$$

Permutation: 31

Zeroes: 16

$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

True

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & \text{lam1} \\ 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) & \frac{\text{lam1} + \text{lam4}}{2} & 0 & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 0 & \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 & -\frac{1}{2\sqrt{2}} \\ 0 & \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 \\ \text{lam1} & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Permutation: 33

Zeros: 16

$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & \text{lam1} \\ 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) & \frac{\text{lam1} + \text{lam3}}{2} & 0 & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 0 & \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 & -\frac{1}{2\sqrt{2}} \\ 0 & \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 \\ \text{lam1} & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Permutation: 35

Zeros: 22

$$\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{\text{lam1} + \text{lam3}}{2} \\ 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & \text{lam1} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \text{lam1} & 0 & 0 & 0 \\ 0 & \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 \\ \frac{\text{lam1} + \text{lam3}}{2} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) \end{pmatrix}$$

Permutation: 36

Zeroes: 22

$$\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{\text{lam1} + \text{lam4}}{2} \\ 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & \text{lam1} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & \text{lam1} & 0 & 0 & 0 \\ 0 & \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 \\ \frac{\text{lam1} + \text{lam4}}{2} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) \end{pmatrix}$$

Permutation: 42

Zeroes: 16

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 & 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \text{lam1} & 0 \\ 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam4}) & \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{2\sqrt{2}} & 0 \\ 0 & \frac{1}{2\sqrt{2}} & \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{2}(-\text{lam1} + \text{lam4}) & -\frac{1}{2\sqrt{2}} & 0 \\ 0 & \text{lam1} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) \end{pmatrix}$$

Permutation: 44

Zeros: 16

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 & 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} \\ 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \text{lam1} & 0 \\ 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam3}) & \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{2\sqrt{2}} & 0 \\ 0 & \frac{1}{2\sqrt{2}} & \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{2}(-\text{lam1} + \text{lam3}) & -\frac{1}{2\sqrt{2}} & 0 \\ 0 & \text{lam1} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) \end{pmatrix}$$

Permutation: 45

Zeros: 22

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 & 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} \\ 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) - \frac{1}{\sqrt{2}} & 0 & \frac{\text{lam1} + \text{lam4}}{2} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & \text{lam1} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & \text{lam1} & 0 & 0 & 0 \\ 0 & \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{\sqrt{2}} & 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 \\ \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) \end{pmatrix}$$

Permutation: 46

Zeroes: 22

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 & 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} \\ 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) - \frac{1}{\sqrt{2}} & 0 & \frac{\text{lam1} + \text{lam3}}{2} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & \text{lam1} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & \text{lam1} & 0 & 0 & 0 \\ 0 & \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{\sqrt{2}} & 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 \\ \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) \end{pmatrix}$$

Permutation: 51

Zeroes: 22

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

True

$$\begin{pmatrix} 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \text{lam1} \\ 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) & \frac{\text{lam1} + \text{lam4}}{2} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 & 0 \\ 0 & \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 \\ \text{lam1} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Permutation: 53

Zeroes: 22

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \text{lam1} \\ 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) & \frac{\text{lam1} + \text{lam3}}{2} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 & 0 \\ 0 & \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 \\ \text{lam1} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Permutation: 55

Zeroes: 16

$$\begin{pmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & \frac{\text{lam1} + \text{lam3}}{2} \\ 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & \text{lam1} & 0 & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 0 & \text{lam1} & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{2\sqrt{2}} \\ 0 & \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 \\ \frac{\text{lam1} + \text{lam3}}{2} & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) \end{pmatrix}$$

Permutation: 56

Zeroes: 16

$$\begin{pmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 0 & \frac{\text{lam1} + \text{lam4}}{2} \\ 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & \text{lam1} & 0 & \frac{1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} & 0 & \text{lam1} & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{2\sqrt{2}} \\ 0 & \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 \\ \frac{\text{lam1} + \text{lam4}}{2} & 0 & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) \end{pmatrix}$$

Permutation: 62

Zeroes: 22

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 & 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \text{lam1} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam4}) & \frac{\text{lam1} + \text{lam4}}{2} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 & 0 \\ 0 & \text{lam1} & 0 & 0 & 0 & 0 \\ \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) \end{pmatrix}$$

Permutation: 64

Zeroes: 22

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 & 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \text{lam1} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam3}) & \frac{\text{lam1} + \text{lam3}}{2} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 & 0 \\ 0 & \text{lam1} & 0 & 0 & 0 & 0 \\ \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) \end{pmatrix}$$

Permutation: 65

Zeroes: 16

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam2} + \text{lam3}) & 0 & 0 & 0 & 0 & \frac{\text{lam2} + \text{lam3}}{2} \\ 0 & \frac{1}{2}(-\text{lam1} + \text{lam4}) & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{\text{lam1} + \text{lam4}}{2} & 0 \\ 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} & \text{lam1} & \frac{1}{2\sqrt{2}} & 0 \\ 0 & \frac{1}{2\sqrt{2}} & \text{lam1} & \frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\ 0 & \frac{\text{lam1} + \text{lam4}}{2} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam4}) & 0 \\ \frac{\text{lam2} + \text{lam3}}{2} & 0 & 0 & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam3}) \end{pmatrix}$$

Permutation: 66

Zeroes: 16

$$\begin{pmatrix} 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & 0 & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

True

$$\begin{pmatrix} \frac{1}{2}(-\text{lam2} + \text{lam4}) & 0 & 0 & 0 & 0 & \frac{\text{lam2} + \text{lam4}}{2} \\ 0 & \frac{1}{2}(-\text{lam1} + \text{lam3}) & -\frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & \frac{\text{lam1} + \text{lam3}}{2} & 0 \\ 0 & -\frac{1}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} & \text{lam1} & \frac{1}{2\sqrt{2}} & 0 \\ 0 & \frac{1}{2\sqrt{2}} & \text{lam1} & \frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}} & 0 \\ 0 & \frac{\text{lam1} + \text{lam3}}{2} & \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} & \frac{1}{2}(-\text{lam1} + \text{lam3}) & 0 \\ \frac{\text{lam2} + \text{lam4}}{2} & 0 & 0 & 0 & 0 & \frac{1}{2}(-\text{lam2} + \text{lam4}) \end{pmatrix}$$