P2: VK1/07 RATKAISUT/PA

2. $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $\quad \det(A-\lambda I) = \begin{vmatrix} -\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix}$

$$= (1-\lambda)(\lambda^2-1) = -(\lambda-1)^2(\lambda+1)$$

$\Rightarrow \lambda_1 = -1$ on 1-kertainen om-arvo

$\lambda_2 = \lambda_3 = 1$ on 2-kertainen

$\lambda_1 = -1$: $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Leftrightarrow \begin{cases} y=0 \\ z=t \\ x=-t \end{cases}$ VAL. $\bar{v}_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\lambda_2 = 1$: $\left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right] \Leftrightarrow \begin{cases} z=t \\ y=s \\ x=t \end{cases}$

$\Rightarrow$ yl. ratk. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\Rightarrow$ voidaan val. $\bar{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\bar{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\Rightarrow \bar{v}_1, \bar{v}_2, \bar{v}_3$ ovat LRT (jopa $\perp$) $\Rightarrow$ A on diagonalisoituva

3. $5x^2 - 2\sqrt{3}\,xy + 3y^2 = \bar{x}^T A \bar{x}$, kun $A = \begin{bmatrix} 5 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{bmatrix}$

Om. arvot: $\begin{vmatrix} 5-\lambda & -\sqrt{3} \\ -\sqrt{3} & 3-\lambda \end{vmatrix} = (5-\lambda)(3-\lambda) - 3 = \lambda^2 - 8\lambda + 12 = 0$

$\Leftrightarrow \lambda = \dfrac{8 \pm \sqrt{16}}{2} = \begin{cases} 2 \\ 6 \end{cases}$ $\Rightarrow$ pääakselimuoto $2x'^2 + 6y'^2 = 12$

$\Leftrightarrow \dfrac{x'^2}{6} + \dfrac{y'^2}{2} = 0$ $\Rightarrow$ puoliakselit $\sqrt{6}, \sqrt{2}$

$\bar{v}_1 = \begin{bmatrix} \sqrt{3} \\ 3 \end{bmatrix}$, $\bar{v}_2 = \begin{bmatrix} -3 \\ \sqrt{3} \end{bmatrix}$, $|\bar{v}_i| = \sqrt{3+9} = 2\sqrt{3}$, $\bar{u}_1 = \dfrac{1}{|\bar{v}_1|}\bar{v}_1 = \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix}$, $\bar{u}_2 = \begin{bmatrix} -\sqrt{3}/2 \\ 1/2 \end{bmatrix}$

$P = \begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$, $P^T = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$ $\Rightarrow \varphi = -\arccos\frac{1}{2} = -\pi/3$

($+\pi/3$ jos vaihd. λ_1, λ_2)

aks. suunnat!