

The Peculiar Thirteen

by the Mællows

In our work, The Peculiar Thirteen, we wanted to explore the mathematics found in music and somehow manage to manifest them in a physical form. We ended up making a ringing sculpture, a bell instrument, tuned to 13-tone Bohlen-Pierce scale.

In order to unveil the mathematics of the Bohlen-Pierce scale, we need first to take a look at the mathematical structures found in our 12-tone chromatic scale, on which majority of western music tradition is built upon.

In 12-tone chromatic scale the tones are repeating in the cycles of octaves, where the harmonic ratio is 2:1. For instance, the frequency of the note A4 is 440 Hz and the tone one octave higher, A5, is 880 Hz, the A6 is 1760 Hz and so forth. This applies in the other direction as well, so the frequency of A3 is 220 Hz and A2 is 110 Hz. The frequency always doubles with each octave, therefore the ratio is 2:1.

As each octave doubles the root note's frequency, and each octave consists of 12 equal semitone intervals, the difference between each semitone can be expressed with the notation $^{12}\sqrt{2} = 2^{1/12}$.

Bohlen-Pierce scale is built quite similarly as the chromatic scale, but the tones, of which there are 13 instead of 12, cycle in the ratio of 3:1 called "tritave". This means that for the root note, for example A = 440 Hz, a tone a tritave higher is 1320 Hz. The difference between each semitone is therefore $^{13}\sqrt{3} = 3^{1/13}$.

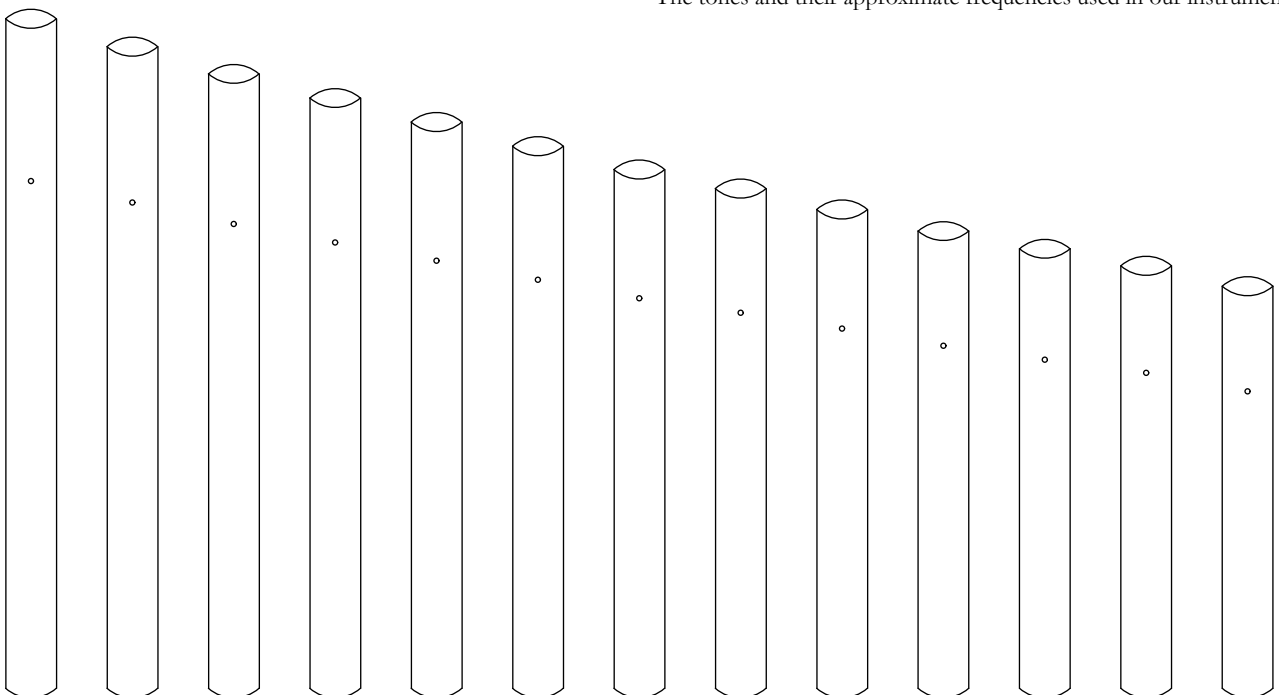
The pipes are tuned by length according to Euler dynamic beam equation in the special case of free vibration to have their 1st resonant mode, or natural frequency, to match the frequencies given in the table. The human hearing complicates things after this as there are multiple resonant modes present in a percussive instrument. In our particular case the higher modes

A	A#	B	C	C#	D	D#	E	F	F#	G	G#	A
440	466	493	523	554	587	622	659	698	739	783	830	880

The notes and their frequencies (Hz) in 12-tone chromatic scale are the following (A4 - A5).

A	A#	B	C	C#	D	E	F	F#	G	H	H#	J	A
440	480	524	568	616	673	735	792	862	942	1025	1109	1223	1320

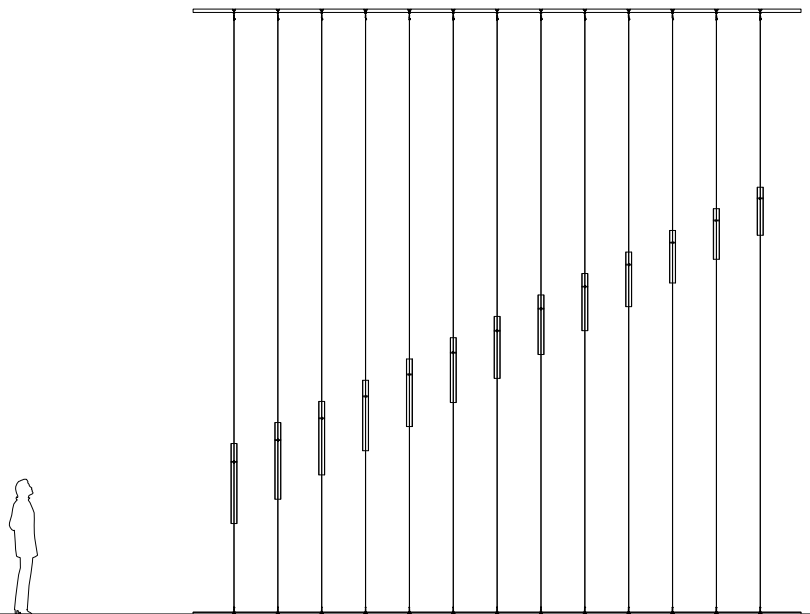
The tones and their approximate frequencies used in our instrument.



have much lower energy than the 1st, but the psycho-acoustic system of an ear combines all the vibrations to a perceived higher tone. Therefore the tones heard are not exactly as described in the previous paragraph but shifted slightly higher in pitch.

We hear music everywhere but rarely we acknowledge the “invisible” mathematical structures behind it. There is no logical or natural reason for these structures but they are only a human-made system, a product of cultural conventions and agreements in the development of western music tradition. Music appears in all cultures around the world, and each culture has developed its own music tradition that often does not fit into the 12-tone chromatic scale.

With the use of unconventional, but equally justified, Bohlen-Pierce-scale we wanted to expose the mathematical foundations of our music system and how music is built only on frequencies, ratios and mathematical functions, without any absolute and correct system.



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