

## Errata & enhancements (newer results)

for PhD Thesis, edition 1, 2002: <https://math.aalto.fi/~kmikkola/research/thesis/> Kalle.Mikkola@iki.fi Errata, version 1.2, 2024-6-29.

No serious errors have been found, but see at least section “A. DANGEROUS ERRORS” below, probably also Section B and section “E. ENHANCEMENTS” at the end.

### Notation

Here “s.r.” means “should read”

“...” denotes a (TeX) math formula

”1. -3” means the 3rd line from the bottom; ”p. 59” means “page 59”; “L2.2.2 means Lemma 2.2.2”.

## A. DANGEROUS ERRORS

(the reader might apply an incorrect result)

!!p. 57, L2.2.2(c2): Add the assumption that  $\mathbb{E} = \mathbb{E}^*$ . (This was missing in [IZ01] and corrected later. Ilya Spitkovsky presented a counter-example and also showed that the corrected result cannot be generalized to  $MTI_d$  (atomic measures).)

p. 287, l. -1: add “ $-1$ ” to the left and replace the lower  $M\tilde{X}$  by  $\tilde{X}\tilde{M}$ .

pp. 325–326: L7.3.6(b2): add the assumption that  $\mathbb{D}$  is DPF-stabilizable (we do not know if this assumption is redundant).

p. 443, Lemma 9.4.3(a1). The statement and proof contain too few details. See [JMAA2007] for a complete statement and proof of Lemma 9.4.3 improved to a major theorem (on interpolation spaces of analytic semigroups).

p. 453, l. 11:  $r < \text{s.r. } -r < \text{l. -9: delete } +2\gamma$ .

p. 507: Proposition 9.10.2(b3): delete the last row. (Thus,  $2^\circ$  of the proof belongs to (b4), where (b1) must be used to justify “by the assumption”.)

p. 539, Lemma 9.14.2(c): “S in one-to-one” s.r. “ $\mathbb{D}$  is  $J$ -coercive”. (However, “ $\Sigma$  is stable” may be removed if one modifies the proof.)

p. 551, l. -1: “ $\ll$ ” s.r. “ $\leq$ ”.

p. 801, l. -13: In (the discrete-time form of) Lemma 4.1.8(e) we may have  $\hat{\mathbb{D}}(0)u_0 = 0$ .

## B. MINOR ERRORS

(same as A., although fairly easily observed as errors)

p. 875 Lemma A.3.1(k1,k2): the conclusions should be exchanged between (k1) and (k2). Indeed,  $U = \ell^2(\mathbf{N})$ ,  $Y = \mathbf{K}$ ,  $Tu := u_n$  is a counter-example to (k1), and  $U = \mathbf{K}$ ,  $Y = \ell^2(\mathbf{N})$ ,  $(Tu)_n = ue_n$  is a counter-example to (k2). (Note that  $\mathbf{K}$  is the scalar field, and  $\mathbf{N}$  the set of natural numbers.)

## C. FAIRLY SAFE ERRORS

(E.g., typos that are rather obvious but appear in results/statements.)

- p. 29: “A-BK” should be “A+BK”
- p. 57: The first line of the proof refers to (c1)(v) only, i.e., there  $L^2$  s.r.  $\pi_+ L^2$ .
- p. 76, Th2.6.4: replace “S=S-S” by “ $\emptyset \neq S = S - S$ ”. (a1): “=  $\mathcal{A}$ ” s.r. “ $\subset \mathcal{A}$ ” and “=  $\tilde{\mathcal{A}}$ ” s.r. “ $\subset \tilde{\mathcal{A}}$ ”. Similarly in its proof on p. 77. N.B. in most cases the Banach algebra  $\mathcal{A}(U)$  is not a C\*-algebra; e.g.,  $\mu := \delta_1 + i\delta_2$  has  $\|\mu * \mu^*\| = 2 < 4 = \|\mu\|^2$ .
- p. 158, l. -4: “ $L^2$ ” s.r. “ $L_\omega^2$ ”.
- p. 170, l. 8: “weak, strong and uniform” s.r. “weak and strong”.
- p. 231, formula (6.142): “ $BLC_L$ ” s.r. “ $BL(I - DL)^{-1}C_s$ ” (restricted to  $H_B$ , whereas  $\text{Dom}(C_L) = \text{Dom}(A_L) \subset H_B$  might be too small for (6.142)).
- p. 251, Theorem 6.7.10(b)(iii) “output” s.r. “SOS”.
- p. 304, Rem7.2.8: remove “ $\hat{\mathbb{Q}}$ ”. On the same line, “ $C$ ” s.r. “ $\Omega$ ”. Three lines lower: “Thus” s.r. “Moreover”.
- p. 311, Th7.2.14(ii)&(ii’): TIC(U) s.r. TIC(Y,U) (ii’): rhs s.r.  $(\tilde{S} + \tilde{U}\tilde{N})^{-1}(\tilde{T} + \tilde{U}\tilde{M})$
- p. 312 l. 3:  $\mathbb{U} \in \text{TIC}$  s.r.  $\mathbb{U} \in \text{TIC}(Y, U)$ . (7.51): rhs s.r.  $(\tilde{S} + \tilde{U}\tilde{N})^{-1}(\tilde{T} + \tilde{U}\tilde{M})$
- p. 370, l. 1-2: Should be “ $\mathbb{K}_{\text{crit}} = -R^{-1}(\pi_+ C \mathbb{B} \tau \pi_+)^* J C_{\text{crit}}$ , i.e.,  $u_{\text{crit}}(x_0) = -R^{-1}(\pi_+ C \mathbb{B} \tau \pi_+)^* J y_{\text{crit}}(x_0)$  for all  $x_0 \in H$ .” (Otherwise the current  $(\pi_+ \mathbb{B} \tau \pi_+)^*$  should be interpreted w.r.t.  $Z_2^S$ , not  $L^2$  inner product, thus making the result useless.)
- In the proof, use, instead, the substitutions  $Z^S := Y^S := L^2(R_+; Y)$ ,  $A := C\mathbb{A}$ ,  $B := C\mathbb{B}\tau = \mathbb{D} - D$ ,  $\tilde{C} := I$  etc.
- p. 374, (8.59):  $\mathbb{B}$  s.r.  $\mathbb{B}\tau$
- p. 442, (f4): delete  $\mathcal{G}$ . (h2): require that  $\beta > 0$  (except in the first claim).
- p. 494, l. -16: “imply” s.r. “imply in cases 1. and 2.”.
- p. 531, Prop 9.13.1(e): the uniqueness of  $S$  (and  $\mathbb{K}$  and  $\mathbb{X}$ ) is again modulo a unit constant operator in  $\mathcal{GB}(U)$ .
- p. 577, l. -5:  $>>$  s.r.  $>$ .
- p. 789, Lemma 13.2.1: “Assume that  $1 \leq p \leq \infty$ .” (e2): (also) the first  $2/p$  should be  $-2/p$ . (Enhancement: multiply both new functions, say  $F$  and  $G$ , by  $2^{1/p}$ ; then they have same  $H^p$  norms as the original ones. Moreover,  $F=g$  iff  $G=f$ . (Multiply  $F$  and divide  $G$  by  $(2\pi)^{-1/p}$  if you want to use the normalized (Haar) measure on the circle.))
- p. 882, Lemma A.3.3(s2):  $\cap$  should be  $\cup$ .
- p. 883, l. -3: “group” s.r. “semigroup”
- p. 884, (B5): “subalgebra” s.r. “subset”, “Banach algebra  $\mathcal{BC}$ ” s.r. “set  $\mathcal{GB} + \mathcal{BC}$ ”.

## D. VERY SAFE ERRORS

(These 1. are almost obvious, or 2. are true but unnecessarily weak, or 3. appear in proofs or other “safe” places. We recommend that the reader corrects to the book the errata listed in Section A., perhaps also those in Section B., whereas those in Section C. are hardly worth the work and only the pages of particular interest should be checked from here. Roughly the same applies to the “Enhancements” section below.)

- p. -1: (the cover of Volume 1/3): “B-ring” should be “B-ring tau”

pp. 3-6 (“Contents”): Add 0–5 to the page numbers in the “Contents” list. (All page numbers outside the “Contents” section are correct, including those in the index.)

p. 20: The second “Aexp” should be “tilde-A”

p. 59: (2.27): “dt” should be “(r)dr”

Next line: “ $\pi_{(-\infty, t)}$ ” should be “ $\pi_{(-\infty, -t)}$ ”

Next line: the second and third “t” s.r. “r”

p. 63, (2.32)–(2.33):  $Z$  should be  $2Z$ . p. 84, l. 9 (3.3): the first “L(“ should be “(L”. l. 16: “for  $t \in A_t$ ” s.r. “for  $u \in A_t$ ”. l. 20 (3.5): the first “=  $\hat{f}$ ” should be “=  $\Lambda \hat{f}$ ”. l. 24:  $Ff$  s.r.  $F\hat{f}$ . l. 25: “ $Ffu = Tfu$  to  $Fg = Tg$ ” s.r. “ $F\hat{f}u = \mathbb{E}\hat{f}u$  to  $F\hat{g} = \mathbb{E}\hat{g}$ ”.

p. 126, l. -5: “ $\mathbb{N}, \mathbb{M}$ ” s.r. “ $\hat{\mathbb{N}}, \hat{\mathbb{M}}$ ” (both twice).

p. 132, l. 6: “and some  $t_k$  smaller than the  $\delta$ ’s in (a) and (b)).” l. 11,13:  $\mathbf{C}$  and  $\mathbf{C}^+$  s.r. “ $\overline{\mathbf{C}^+}$ ”. l. 16: add “ $k \neq n$ ” before “, so that”. l. 18: both 1’s should be  $\sqrt{2\pi}$ .

p. 133, l. 1: “ $H$ ” s.r. “ $H^2$ ”.

p. 140: “[GL-Crit]” s.r. “[GL73b]”.

p. 147, formula (5.13): “ $I + g$ ” s.r. “ $I + g^*$ ”.

p. 148, (5.13): add \* to the end (after g).

p. 159, (6.13):  $\|\tau^t \mathbb{B}u\|_H$  s.r.  $\|\mathbb{B}\tau^t u\|_{L_\omega^2}$ . On the previous line,  $\pi_+ \tau \mathbb{B}$  s.r.  $\pi_+ \tau^t \mathbb{B}$ ; still 4 lines earlier: “so that” s.r. “define  $H := \mathbb{B}[L_\omega^2]$  and” (In a similar manner, from any  $\omega$ -stable WPLS one obtains an exactly  $\omega$ -reachable  $\omega$ -stable WPLS by replacing the state space by  $\mathbb{B}[L_\omega^2]$  and by increasing the norm as in the proof of Lemma 6.1.7.)

p. 169: D6.2.3 “and  $\mathbb{D} \in B(U, Y)$ ” s.r. “and  $D \in B(U, Y)$ ”.

p. 179, Proof: “(a)&(b)” s.r. “(a1)-(b2)”. On the same line, 4.2 and 4.5 s.r. 5.4 and 5.5.

p. 185, Prop6.3.4(a1): “UVR,  $\mathbb{D}u :=$ ” po. “UVR,  $\mathbb{D}u =$ ”.

p. 189, below Example 6.3.7: “ $\Sigma$  divided by” s.r. “ $\mathbb{C}$  and  $\mathbb{D}$  divided by”.

p. 191, l. 6:  $\mathcal{C}$  s.r.  $C_c$ .

p. 197, l. -2:  $\mathbb{D} \in L^2$  should be  $\mathbb{D}u \in L^2$ .

p. 257: In (6.191), replace  $[L \ 0]$  by  $[0 \ I]$ . Remove the preceding sentence (including (6.190)).

p. 301, formula (7.33) “ $\mathbb{M}$ ” s.r. “ $\tilde{\mathbb{M}}$ ” (twice).

p. 326, l. -15:  $\tilde{\Sigma} = [0, 0; 0, 0]$  s.r.  $\tilde{\Sigma} = [e^-, 0; 0, 0]$ .

p. 331, Th7.3.12(c):  $B$  s.r.  $B_1$ ,  $C^*$  s.r.  $C_2^*$ .

p. 364, l. 15:  $L^2$  s.r.  $L_\varepsilon^2$ .

p. 377, l. 15: “second and third” s.r. “third and fourth”. l. 11: Add superscript  $d$  to both sides of the equation.

p. 380, l. -12: “implies all” s.r. “is implied by any”.

p. 429, (c3): (iv’) s.r. (iv).

pp. 443-444: the proof is insufficient. See [JMAA2007] for a complete statement and proof of Lemma 9.4.3 (upgraded to a major theorem).

pp. 443-444: 2.2.13 s.r. 2.2.15.

p. 444, l. 22:  $L\mathbf{A}x$  should read  $\Lambda x$ .

p. 447, l. -5: Add “for any  $\omega_0 > \omega_A$ ”.

p. 473, Prop 9.8.7(c3): Replace  $\mathcal{U}_{out}$  by  $\mathcal{U}_*$  (or vice versa).

p. 481, (9.128): “)” s.r. “)<sub>w</sub>”.

- p. 512, l. -7: add “if  $(\cdot - A)^{-1}B$  is replaced by  $\widehat{\mathbb{B}\tau}$ ” (these need not be equal on  $\mathbf{C}_{\omega_A}^-$ ).
- l. -5: “ $\tau^t$ ” s.r. “ $\tau^{-t}$ ”; latter “ $v$ ” should be “ $u$ ”
- l. -4: latter “ $(s - A)^{-1}Bu_0$ ” s.r. “ $(z - A)^{-1}Bv_0$ ” (l. -3: “ $t$ ” s.r. “ $r$ ”).
- p. 527, l. 7: add  $K_{ring}F_{ring}$  below  $0I$ .
- p. 532, l. 7:  $JK$  s.r.  $SK$ .
- p. 538, l. 12: s.r.  $Y = H \times H$ .
- p. 545, l. -13:  $iR$  s.r.  $\partial D$ .
- p. 580, l. -2:  $u$  s.r.  $u_n$  (twice).
- p. 642, l. 5:  $M_{22}$  s.r.  $M_{22}^{-1}$ . l. 9 & 10:  $\gamma^2$  s.r.  $\gamma^2\|w\|_2^2$
- p. 785, (13.21): p should read 2. (d), 2°: The last sentence should read “This and (c2) imply (13.17).”
- p. 795, l. 9:  $N$  s.r.  $-N$ .
- p. 798, l. -7: “; take ...” should read “.  $\mathbb{B}$  is  $r$ -stable, because  $\ell_r^2 \ni f \mapsto f(0) \in H$  is bounded.”
- p. 799, l. 5:  $ti_r$  s.r.  $ti_s$ .
- p. 808, l. -2:  $e^r$  s.r.  $e^\alpha$ . l. -2:  $\Delta^S$  s.r.  $\Delta^{\ell^2}$ .
- p. 809, l. 2, 7, 8:  $\Delta^S$  s.r.  $\Delta^{\ell^2}$ . (similarly in the proof, p. 811).
- p. 811, two lines below (13.77): the latter  $\Delta^S x$  s.r.  $\Delta^{\ell^2} u$ .
- p. 850, l. 8: multiply the right column of the matrix by -1.
- p. 882, (A3):  $\notin$  s.r.  $\in$ .
- p. 885, (N4)&(N5): “ $U$ ” s.r. “ $B_1$ ” and “ $Y$ ” s.r. “ $B_2$ ”.
- p. 886, (R2): (Both) “reflexive” s.r. “separable”. l. -8: “, then” s.r. “is onto, then”.
- p. 898, (A.33):  $X$  s.r.  $X \cap H$ .
- p. 904, l. -9: replace  $M_r + 1$  by  $M_r + M'_r$  for a suitable constant  $M'_r$ .
- p. 911, l. 9: “are measurable” s.r. “are disjoint and measurable”.
- p. 921, (B.9): the latter  $\Omega$  s.r.  $\Omega \setminus K$ .
- p. 944, l. 1: replace “ $-F(t)+$ ” by “ $-F(t) = \int_a^{b(t)} [f(t+h, s) - f(t, s)]ds + \int_{b(t)}^{b(t+h)} [f(t+h, s) - f(t, b(t))]ds$ ”. l. 3: replace “ $ds-$ ” by “ $ds - h$ ”.
- p. 971, (d’): the latter  $f$ ’ should have a hat ( $\hat{f}$ )
- p. 983, D.1.24(b): replace “a.e.  $r \in E$ ” by “any point  $ir$  of metric density 1 of  $E$  (hence for a.e.  $ir \in E$ )”. (This is just an enhancement, but it was used in the proof of L4.1.8.)
- p. 984, (D.52): “=” s.r. “ $\leq$ ”.
- p. 987, (E.1):  $L^1 \cap L^\infty$  s.r.  $L^1 + L^\infty$ .
- p. 1003, l. 4: add “and  $H : Q \rightarrow \mathcal{B}(B, B_2)$ ”. l. -8:  $LF(q)x$  s.r.  $(LFx)(q)$ .
- p. 1019, l. 9: “Lemma F.1.6” s.r. “Closed-Graph Theorem”.
- p. 1021, l. 4: “ $\hat{\mathbb{D}}$ ” s.r. “ $\hat{F}$ ”. l. 7: The first  $U$  s.r.  $\mathcal{B}(U)$ .
- p. 1022, l. 9:  $U, Y$  s.r.  $Y, U$
- p. 1030: [LR] = Peter Lancaster and Leiba Rodman, Algebraic Riccati Equations, 1995.

## E. ENHANCEMENTS (newer results)

The papers published by the author after this PhD thesis enhance remarkably some results on state-feedback stabilizability and output-feedback stabilizability, [quasi]-coprime factorizations and boundary functions. This mostly affects Chapters 7, 10, 6, 9, 4 and 3, in that order. For details, see papers linked at (also the CDC-ECC'05 paper (or slides), as a corresponding journal paper has not yet been written): <https://math.aalto.fi/~kmikkola/research/index.html>

Some examples are given below:

Chapter 6: A system is **optimizable iff it is exponentially stabilizable**.

Similarly, **for each  $x_0$  there exists  $u \in L^2$  such that  $y \in L^2$  iff the system is output-stabilizable**, or equivalently, **iff the system is r.c.-SOS-stabilizable**. The latter means that there exists a state-feedback that is well-posed (not just in its closed-loop but also in its open-loop form!), the corresponding closed-loop output and I/O operators are stable, and the closed-loop input-to-output and feedback-to-output operators are **right coprime**. [IEOT2007]

Numerous other new connections between different forms of stabilization and detection are given (in fact, very many conditions are equivalent, as shown in [CDC-ECC2005]). Any function having a right factorization has a q.r.c.f.

Chapter 7: An I/O map has a d.c.f. iff it has a stabilizing controller with internal loop, equivalently, iff it has a r.c.f, iff it has an output-stabilizable and input-detectable realization (this last condition is due to Curtain and Opmeer). (Several further equivalent conditions and related results are given, also for the dynamic stabilizability of a system (not merely of the I/O map).) [SIAM2007] [CDC-ECC2005]

p. 333, Prop7.3.14: “and  $\mathbb{D}_{21}$  has a d.c.f.” is redundant, by [SIAM2007].

Chapter 9, Lemma 9.4.3: See [JMAA2007] for a complete statement and proof of Lemma 9.4.3 improved to a major theorem (on interpolation spaces of analytic semigroups). (It also contains other relevant results and the application in the title.)

Volume 2, several places: “Assume that  $Z^s$  is reflexive” (and hence outruling  $U_{str}, U_{sta}$ ) is unnecessary, because Lemma 8.2.3 can be proved by assuming only that  $Y^s$  is reflexive (thus we can remove  $U$  and  $Z^s$  from Hypothesis 8.2.2), the only exception being that the Banach space  $\mathcal{U}(0)$  need not be reflexive (in Lemma 8.2.3(b)); however, when  $D$  is  $J$ -coercive, then  $\mathcal{U}(0)$  is TVS isomorphic to  $Y^s$ , hence reflexive).

Volume 2: Assume that  $U_*^* = U_{out}$  or that  $U_*^* = U_{exp}$  and that the system is positively  $J$ -coercive. Then there is a  $J$ -critical state-feedback pair over  $U_{out}$  iff  $U_*(x_0) \neq \emptyset$  for all  $x_0 \in H$ .

(In particular, a WPLS is optimizable iff it is exponentially stabilizable.)

**Real solutions:** If the system is real (as opposed to strictly complex), then most main solutions provided in my math papers (and in the thesis) are real. For example, if a system is real and stabilizable, then our formulae for stabilizing state-feedback operators yield real operators (and hence also the outputs become real if inputs and initial states are real). This applies also to stabilizing output feedback and to almost all other problems solved in the papers. [SIAM2012]

## References

You can read all these freely at

<https://math.aalto.fi/~kmikkola/research/index.html>

[CDC-ECC2005] Coprime factorizations and stabilization of infinite-dimensional linear systems. Kalle M. Mikkola. Proceedings of CDC-ECC2005

[JMAA2007] The Hilbert-Schmidt property of feedback operators. Ruth F. Curtain, Kalle M. Mikkola and Amol J. Sasane. Journal of Mathematical Analysis and Applications, 329 (2), pp. 1145-1160, 2007

[IEOT2007] State-Feedback Stabilization of Well-Posed Linear Systems. Kalle M. Mikkola. Integral Equations and Operator Theory 55 (2), pp. 249-271, 2006.

[SIAM2007] Coprime factorization and dynamic stabilization of transfer functions. Kalle M. Mikkola. SIAM Journal on Control and Optimization, 45 (6), pp. 1988-2010, 2007.

[SIAM2012] Real solutions to control, approximation and factorization problems. Kalle M. Mikkola. SIAM J. Control Optim., 50(3), 1071-1086, 2012.

The slides of [CDC-ECC2005] are strongly recommended, even though the conference paper contains a bit more. (Also a newer two-paper manuscript with additional results exists, but I had no time to finalize it before new responsibilities.)