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Volume 1/3 — Time-Invariant Operators & WPLSs

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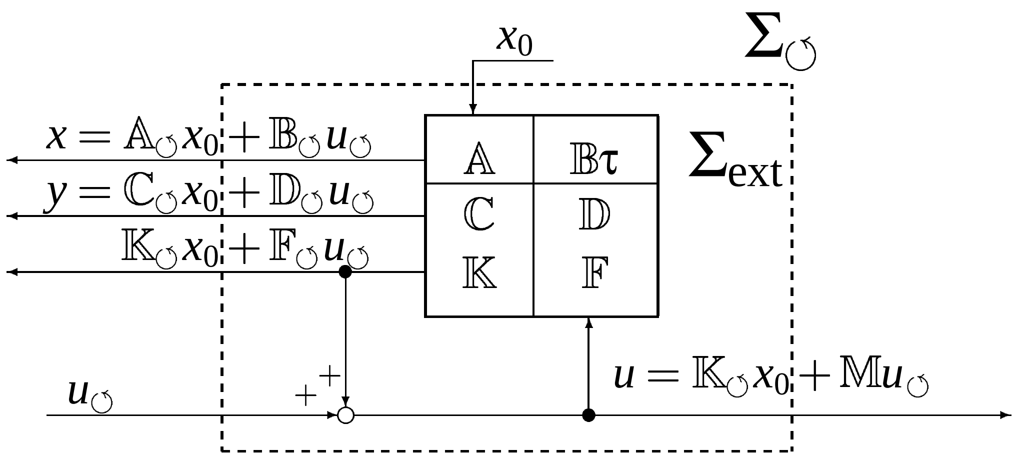

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Infinite-Dimensional Linear Systems, Optimal Control and Algebraic Riccati Equations

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Kalle Mikkola: *Infinite-Dimensional Linear Systems, Optimal Control and Algebraic Riccati Equations*; Helsinki University of Technology Institute of Mathematics Research Reports A452 (2002).

Abstract: *In this monograph, we solve rather general linear, infinite-dimensional, time-invariant control problems, including the* H^∞ *and LQR problems, in terms of algebraic Riccati equations and of spectral or coprime factorizations. We work in the class of (weakly regular) well-posed linear systems (WPLSs) in the sense of G. Weiss and D. Salamon.*
Moreover, we develop the required theories, also of independent interest, on WPLSs, time-invariant operators, transfer and boundary functions, factorizations and Riccati equations. Finally, we present the corresponding theories and results also for discrete-time systems.

AMS subject classifications: 42A45, 46E40, 46G12, 47A68, 49J27, 49N10, 49N35, 93-02, 93A10, 93B36, 93B52, 93C05, 93C55, 93D15

Keywords: suboptimal H-infinity control, standard H-infinity problem, measurement feedback problem; H-infinity full information control problem, state feedback problem; Nehari problem; LQC, LQR control, H2 problem, minimization; bounded real lemma, positive real lemma; dynamic stabilization, controller with internal loop, strong stabilization, exponential stabilization, optimizability; canonical factorization, (J,S)-spectral factorization, (J,S)-inner coprime factorization, generalized factorization, J-lossless factorization; compatible operators, weakly regular well-posed linear systems, distributed parameter systems; continuous-time, discrete-time; infinite-horizon; time-invariant operators, Toeplitz operators, Wiener class, equally-spaced delays, Popov function; transfer functions, H-infinity boundary functions, Fourier multipliers; Bochner integral, strongly measurable functions, strong Lp spaces; Laplace transform, Fourier transform

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Preface

For H^∞, H^2, LQR and several other linear time-invariant control problems, it is well known that the existences of

(I) a solution of that control problem,

(II) a corresponding coprime or spectral factorization and

(III) a stabilizing solution of the corresponding Riccati equation

are, roughly speaking, all equivalent in the finite-dimensional setting, and from one of them the others can be computed. Therefore, control problems are often solved by computing the solutions of the corresponding Riccati equations.

These results have been extended to infinite-dimensional (semigroup) control systems with bounded input and output operators, and in the eighties and early nineties also to the larger class of Pritchard–Salamon systems and to certain other special cases of our setting.

The main purpose of this monograph is to generalize these results to infinite-dimensional (weakly regular) well-posed linear systems (WPLSs) in the sense of G. Weiss. This is done in Chapters 8–12; see pp. 21–24 for an introduction to WPLSs.

We also develop corresponding discrete-time results (Chapters 13–15 and Sections 11.5 and 12.2), WPLS theory (Chapters 6–7, including regularity, state feedback, output injection and dynamic feedback), and an extensive theory of independent interest for time-invariant operators ("Toeplitz operators") and some of their subclasses (such as extended Callier–Desoer classes and other convolutions with measures), transfer and boundary functions and spectral and coprime factorization (Chapters 2–5 and Sections 6.4–6.5). A more detailed description of some of the main results of this monograph and some historical remarks are provided in Sections 1.1 and 1.2 and in the "notes" parts of each section.

WPLSs cover all linear time-invariant systems that map the initial state and input continuously to the state and output (with inputs and outputs in L^2_{loc}; see pp. 21); in particular all settings mentioned above are covered. Moreover, any transfer function that is bounded and holomorphic on some right half-plane (i.e., that is *well-posed* or *proper*) has a WPLS realization. The input and output operators of a WPLS may be as unbounded as for Pritchard–Salamon systems *independently* of each other as long as the transfer function is well posed, thus allowing roughly twice as much irregularity.

Weak regularity means the existence of a feedthrough operator in a very weak sense; an equivalent condition is that the transfer function has a limit at infinity along the positive real axis. In particular, all I/O maps whose impulse response is a (uniform, strong or weak) L^p function plus some delays (or any vector-valued measure) and several others are weakly regular.

Much of our theory on WPLSs cover the general case, but Riccati equations cannot be defined without feedthrough operators (except in a very weak sense, as in Section 9.7).

We generally allow the input, state and output spaces of WPLSs to be Hilbert spaces of arbitrary dimensions, and some results are given even in a Banach space setting. In addition to exponentially stabilizing controllers and exponentially stabilizing solutions of Riccati equations, we also study stabilizing and strongly stabilizing ones; part of these results may be new even for finite-dimensional systems.

During the last four decades, the literature has become abundant in infinite-dimensional (or *distributed*) systems arising in physics, engineering, economics, mechanics, environmental modeling, biomedical engineering, evolution dynamics, geophysics and other sciences, and the systems can represent semiconductor devices, animal populations, fluid dynamics, microwave circuits, vibration of strings or membranes, heat diffusion, computer hard discs, CD players and many other devices.

For some particular systems and problems, there are now rather mature theories. The purpose of this monograph is to solve the problems in a very general, unifying framework — in the framework of WPLSs.

Our presentation is abstract and theory-oriented; nevertheless, many of our results can be understood without the functional analytic knowledge provided by the appendices. The book is rather self-contained and it can be read without any prior knowledge in system or control theory, although experts are considered as the main audience and some proofs may be demanding.

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Professor Ilya Spitkovsky has kindly pointed me to several important results on spectral factorization and Irena Lasiecka and Roberto Triggiani to many results obtained by their school. I have also had inspiring discussions with professors Ruth Curtain, George Weiss and Hans Zwart, doctors Marko Huhtanen and Jarmo Malinen, and others. Part of this work was written with the support of the Academy of Sciences, the Graduate School of Analysis and the Finnish Cultural Foundation. Finally, I wish to thank my friends and loved ones, for making this world such a great place to live in.

Kalle Mikkola

Twice or thrice had I loved thee
Before I knew thy face or name.
So in a voice, so in a shapeless flame,
Angels affect us oft, and worshipped be.
— John Donne (1571–1631)