```
In[1]:= SetDirectory["~/KappaLib/"];
        << kappaLib-1.1.m
        << helper.m

        KappaLib v1.1

        Loading helper.m..
```

▪ Metaclass III:

```
In[4]:= vars = {x0, x1, x2, x3};
```

$$
In[5]:= \texttt{kappa = emMatrixToKappa}\left[
\begin{pmatrix}
a1 & -b1 & 0 & 0 & 0 & 0 \\
b1 & a1 & 0 & 0 & 0 & 0 \\
1 & 0 & a1 & 0 & 0 & -b1 \\
0 & 0 & 0 & a1 & b1 & 1 \\
0 & 0 & 1 & -b1 & a1 & 0 \\
0 & 1 & b1 & 0 & 0 & a1
\end{pmatrix}
\right];
$$

```
In[6]:= fr = emKappaToFresnel[kappa, vars];
        Collect[FullSimplify[Expand[fr]], vars]
```

$$
Out[7]= b1^3\, x1^4 + 2\, b1^3\, x1^2\, x2^2 + b1^3\, x2^4 + x0^3\,(2\, x2 - 2\, b1\, x3) +
$$
$$
x0\,(-4\, b1^3\, x1^2\, x3 - 4\, b1^3\, x2^2\, x3) + x0^2\,(-3\, b1\, x1^2 - 3\, b1\, x2^2 + 4\, b1^3\, x3^2)
$$

▪ We assume that the Fresnel polynomial factorises:

```
In[8]:= A = Table[ToExpression["A" <> ToString[Min[{i, j}]] <> ToString[Max[{i, j}]]],
            {i, 0, 3}, {j, 0, 3}];
        B = Table[ToExpression["B" <> ToString[Min[{i, j}]] <> ToString[Max[{i, j}]]],
            {i, 0, 3}, {j, 0, 3}];
        A // MatrixForm
        B // MatrixForm
        factorised = (vars.A.vars) (vars.B.vars);
```

$$
Out[10]//MatrixForm=
\begin{pmatrix}
A00 & A01 & A02 & A03 \\
A01 & A11 & A12 & A13 \\
A02 & A12 & A22 & A23 \\
A03 & A13 & A23 & A33
\end{pmatrix}
$$

$$
Out[11]//MatrixForm=
\begin{pmatrix}
B00 & B01 & B02 & B03 \\
B01 & B11 & B12 & B13 \\
B02 & B12 & B22 & B23 \\
B03 & B13 & B23 & B33
\end{pmatrix}
$$

▪ Compute Gröbner basis constraints on A and B

```
In[13]:= cons = Union[Flatten[CoefficientList[fr - factorised, vars]]];
```

```
In[14]:= gb = GroebnerBasis[cons, Variables[cons]]; // Timing
```

```
Out[14]= {34.0468, Null}
```

```
In[15]:= Length[gb]
         Take[emSort[gb], 5] // show
```

```
Out[15]= 346
```

$$
Out[16]//MatrixForm=
\begin{pmatrix}
1 & : & b1^4 \\
2 & : & A01^3 \\
3 & : & A12^3 \\
4 & : & A13^2 \\
5 & : & A22^4
\end{pmatrix}
$$

▪ Equation (1) contradicts b1 != 0.