Plot contact forms from TE_11 and TM_11 fields. We only plot the +-fields as the --fields are contact forms if and only if the +-fields are contact forms. See notebook: contactFromTETM.nb

```
(* before evaluating the below commands, run notebook contactFromTETM.nb
  Note also that we assume that t=0.
*)
ComplexExpand[Re[BelTE[x, y, z, m, n, +1] /.
   {ϵ → 1, μ → 1, a → 1, b → 1, β → 1, ω → 1, m → 1, n → 1, t → 0}]]
ComplexExpand[Re[BelTM[x, y, z, m, n, +1] /.
   {ϵ → 1, μ → 1, a → 1, b → 1, β → 1, ω → 1, m → 1, n → 1, t → 0}]]
```

$$\left\{\frac{\text{Cos}[\pi y]\,\text{Cos}[z]\,\text{Sin}[\pi x]}{4\pi\sqrt{1+2\pi^2}}+\frac{\text{Cos}[\pi x]\,\text{Sin}[\pi y]\,\text{Sin}[z]}{4\pi},\right.$$

$$\left.\frac{\text{Cos}[\pi x]\,\text{Cos}[z]\,\text{Sin}[\pi y]}{4\pi\sqrt{1+2\pi^2}}-\frac{\text{Cos}[\pi y]\,\text{Sin}[\pi x]\,\text{Sin}[z]}{4\pi},-\frac{\text{Cos}[\pi x]\,\text{Cos}[\pi y]\,\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\}$$

$$\left\{\frac{\text{Cos}[\pi y]\,\text{Cos}[z]\,\text{Sin}[\pi x]}{4\pi\sqrt{1+2\pi^2}}+\frac{\pi\,\text{Cos}[\pi y]\,\text{Cos}[z]\,\text{Sin}[\pi x]}{2\sqrt{1+2\pi^2}}-\frac{\text{Cos}[\pi x]\,\text{Sin}[\pi y]\,\text{Sin}[z]}{4\pi},\right.$$

$$-\frac{\text{Cos}[\pi x]\,\text{Cos}[z]\,\text{Sin}[\pi y]}{4\pi\sqrt{1+2\pi^2}}-\frac{\pi\,\text{Cos}[\pi x]\,\text{Cos}[z]\,\text{Sin}[\pi y]}{2\sqrt{1+2\pi^2}}-\frac{\text{Cos}[\pi y]\,\text{Sin}[\pi x]\,\text{Sin}[z]}{4\pi},$$

$$\left.\frac{1}{2}\,\text{Cos}[z]\,\text{Sin}[\pi x]\,\text{Sin}[\pi y]\right\}$$

$$\text{TEplus}[x\_, y\_, z\_] := \left\{\frac{\text{Cos}[\pi y]\,\text{Cos}[z]\,\text{Sin}[\pi x]}{4\pi\sqrt{1+2\pi^2}}+\frac{\text{Cos}[\pi x]\,\text{Sin}[\pi y]\,\text{Sin}[z]}{4\pi},\right.$$

$$\left.\frac{\text{Cos}[\pi x]\,\text{Cos}[z]\,\text{Sin}[\pi y]}{4\pi\sqrt{1+2\pi^2}}-\frac{\text{Cos}[\pi y]\,\text{Sin}[\pi x]\,\text{Sin}[z]}{4\pi},-\frac{\text{Cos}[\pi x]\,\text{Cos}[\pi y]\,\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\}$$

$$\text{TMplus}[x\_, y\_, z\_] :=$$

$$\left\{\frac{\text{Cos}[\pi y]\,\text{Cos}[z]\,\text{Sin}[\pi x]}{4\pi\sqrt{1+2\pi^2}}+\frac{\pi\,\text{Cos}[\pi y]\,\text{Cos}[z]\,\text{Sin}[\pi x]}{2\sqrt{1+2\pi^2}}-\frac{\text{Cos}[\pi x]\,\text{Sin}[\pi y]\,\text{Sin}[z]}{4\pi},\right.$$

$$-\frac{\text{Cos}[\pi x]\,\text{Cos}[z]\,\text{Sin}[\pi y]}{4\pi\sqrt{1+2\pi^2}}-\frac{\pi\,\text{Cos}[\pi x]\,\text{Cos}[z]\,\text{Sin}[\pi y]}{2\sqrt{1+2\pi^2}}-\frac{\text{Cos}[\pi y]\,\text{Sin}[\pi x]\,\text{Sin}[z]}{4\pi},$$

$$\left.\frac{1}{2}\,\text{Cos}[z]\,\text{Sin}[\pi x]\,\text{Sin}[\pi y]\right\}$$

```
(* Load planefield plotter. If this generates fatal error messages, quit kernel,
rerun definitions of TEplus and TMplus and rerun the below commands.
 *)
SetDirectory["~/writing/WIP/contact/plots/"];
<< CSPlotter.m
```

General::obspkg :
  Graphics`Shapes` is now obsolete. The legacy version being loaded may conflict with current Mathematica functionality. See the Compatibility Guide for updating information. ≫

ListSurfacePlot3D::shdw :
  Symbol ListSurfacePlot3D appears in multiple contexts {Graphics`Graphics3D`, System`}; definitions in context Graphics`Graphics3D` may shadow or be shadowed by other definitions. ≫

Histogram3D::shdw :
  Symbol Histogram3D appears in multiple contexts {Graphics`Graphics3D`, System`}; definitions in context Graphics`Graphics3D` may shadow or be shadowed by other definitions. ≫

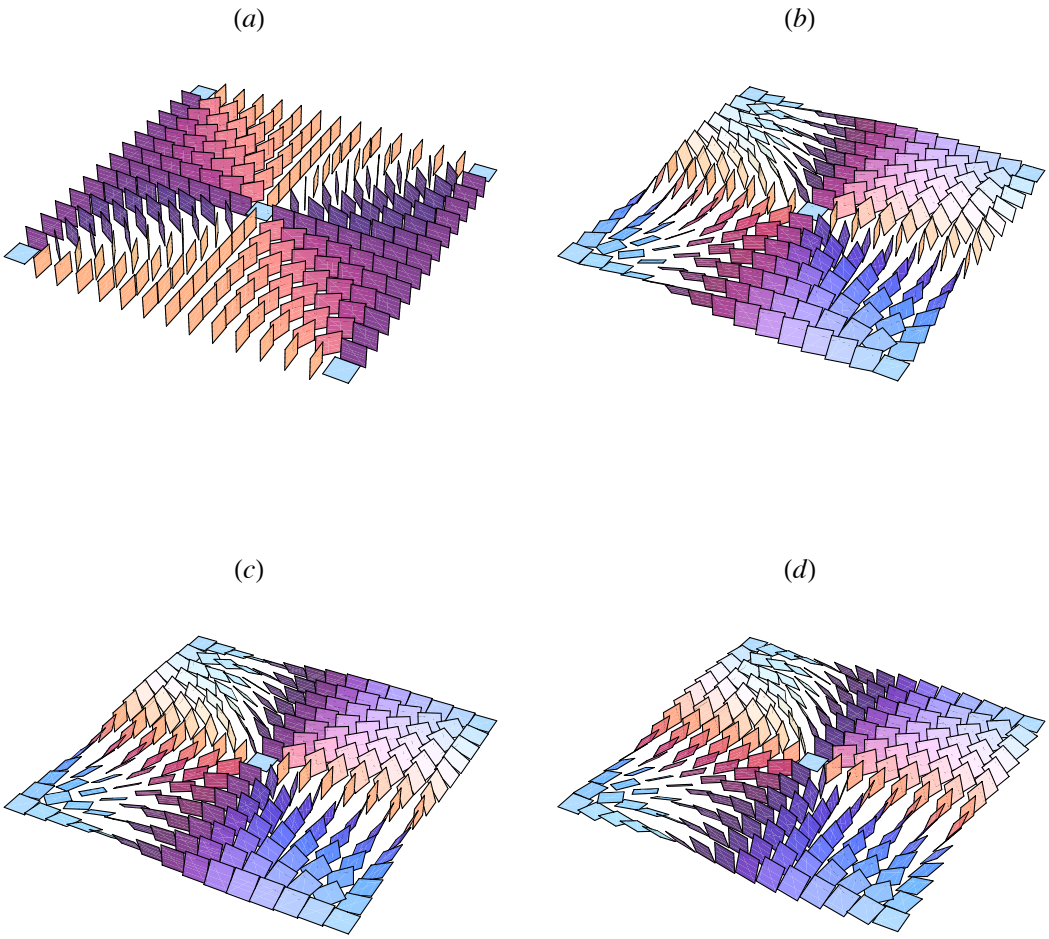
BarChart3D::shdw :
  Symbol BarChart3D appears in multiple contexts {Graphics`Graphics3D`, System`}; definitions in context Graphics`Graphics3D` may shadow or be shadowed by other definitions. ≫

General::newpkg :
  Calculus`VectorAnalysis` is now available as the Vector Analysis Package. See the Compatibility Guide for updating information. ≫

```
xmin = 0; xmax = 1; dx = (xmax - xmin) / 14;
ymin = 0; ymax = 1; dy = (ymax - ymin) / 14;
scale = dy;
```

```
plotTE[z_, lbl_] := Module[{t},
  t = Table[contactelement[{x, y, 0}, TEplus[x, y, z], scale],
    {x, xmin, xmax, dx}, {y, ymin, ymax, dy}];
  Show[t, Boxed → False, PlotLabel → Style[lbl <> "\n", Italic, 13]]
  (* Labeled[t,Style[lbl,Italic,17]]*)
 ]
plotTM[z_, lbl_] := Module[{t},
  t = Table[contactelement[{x, y, 0}, TMplus[x, y, z], scale],
    {x, xmin, xmax, dx}, {y, ymin, ymax, dy}];
  Show[t, Boxed → False, PlotLabel → Style[lbl <> "\n", Italic, 13]]
  (*Labeled[t,Text[Style[lbl,Italic,17]]]*)
 ]
```

```
e0 = plotTE[Pi 0 / 4, "(a)"];
e1 = plotTE[ Pi 1 / 4, "(b)"];
e2 = plotTE[ Pi 2 / 4, "(c)"];
e3 = plotTE[ Pi 3 / 4, "(d)"];
TEarr = GraphicsArray[{{e0, e1}, {e2, e3}}]
```




```
Export["~/writing/WIP/contact/TE11plus.pdf", TEarr]
```

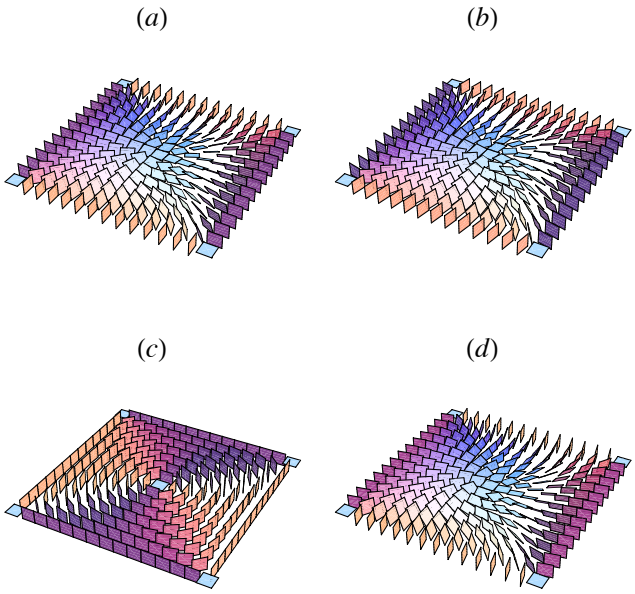
```
~/writing/WIP/contact/TE11plus.pdf
```

```
TEplus[1 / 2, 1 / 2, z]
```

```
{0, 0, 0}
```

TM

```
e0 = plotTM[Pi 0 / 4, "(a)"];
e1 = plotTM[ Pi 1 / 4, "(b)"];
e2 = plotTM[ Pi 2 / 4, "(c)"];
e3 = plotTM[ Pi 3 / 4, "(d)"];
TMarr = GraphicsArray[{
    {e0, e1},
    {e2, e3}
    (*{e4,e5}*)}
 ]
```




```
Export["~/writing/WIP/contact/TM11plus.pdf", TMarr]
```

```
~/writing/WIP/contact/TM11plus.pdf
```

```
(* There does not seem to be a way to place labels under a 3D graphics
 object. Therefore I manually moved the labels to under the objects using Photoshop. *)
```

- **Periodicity:**

```
TEplus[x, y, z] + TEplus[x, y, z + Pi]
TMplus[x, y, z] + TMplus[x, y, z + Pi]
```

```
{0, 0, 0}
```

```
{0, 0, 0}
```

- **Type of failure: check that the contact condition always fails due to the fact the the field vanishes. This follows since both fields are Beltrami fields.**

```
TEplus[1 / 2, 1 / 2, z]
{TEplus[0, 0, z], TEplus[0, 1, z], TEplus[1, 0, z], TEplus[1, 1, z]}
```

```
{0, 0, 0}
```

$$\left\{\left\{0, 0, -\frac{\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\}, \left\{0, 0, \frac{\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\}, \left\{0, 0, \frac{\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\}, \left\{0, 0, -\frac{\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\}\right\}$$

```
TMplus[1 / 2, 1 / 2, z]
{TEplus[0, 0, z], TEplus[0, 1, z], TEplus[1, 0, z], TEplus[1, 1, z]}
```

$$\left\{0,\ 0,\ \frac{\text{Cos}[z]}{2}\right\}$$

$$\left\{\left\{0,\ 0,\ -\frac{\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\},\ \left\{0,\ 0,\ \frac{\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\},\ \left\{0,\ 0,\ \frac{\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\},\ \left\{0,\ 0,\ -\frac{\text{Sin}[z]}{2\sqrt{1+2\pi^2}}\right\}\right\}$$