- **Computations related proof of Theorem 4.3**

- **Claim 1: If F^(ij) xi_ij = 0, then xi is of the form**

$$\begin{pmatrix} 0 & AA & BB & CC \\ -AA & 0 & DD & -BB \\ -BB & -DD & 0 & EE \\ -CC & BB & -EE & 0 \end{pmatrix}$$

```
(* In general xi is of the form *)
XI = {{0, AA, BB, CC},
   {-AA, 0, DD, -GG},
   {-BB, -DD, 0, EE},
   {-CC, GG, -EE, 0}};
XI // MatrixForm
XI + Transpose[XI]
```

$$\begin{pmatrix} 0 & AA & BB & CC \\ -AA & 0 & DD & -GG \\ -BB & -DD & 0 & EE \\ -CC & GG & -EE & 0 \end{pmatrix}$$

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

```
Fmat = {{0, 0, 1, 0},
   {0, 0, 0, 1},
   {-1, 0, 0, 0},
   {0, -1, 0, 0}};
Fmat // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

```
Simplify[Sum[Fmat[[i]][[j]] XI[[i]][[j]], {i, 1, 4}, {j, 1, 4}] == 0]
```

```
BB == GG
```

```
(* Thus: if F^(ij) xi_ij=0, then BB=GG and the claim follows *)
```

- **Claim 2: If xi is as above and**

  ξ_pi F^ij G_jq = ξ_qi F^ij G_jp

  then xi=0

```
(* By the above argument we assume that xi is of the form *)
XI = {{0, AA, BB, CC},
   {-AA, 0, DD, -BB},
   {-BB, -DD, 0, EE},
   {-CC, BB, -EE, 0}};
XI // MatrixForm
XI + Transpose[XI]
```

$$\begin{pmatrix} 0 & AA & BB & CC \\ -AA & 0 & DD & -BB \\ -BB & -DD & 0 & EE \\ -CC & BB & -EE & 0 \end{pmatrix}$$

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

```
Gmat = {{0, G12, G13, G14},
   {-G12, 0, G23, G24},
   {-G13, -G23, 0, G34},
   {-G14, -G24, -G34, 0}};
Gmat // MatrixForm
Gmat + Transpose[Gmat]
```

$$\begin{pmatrix} 0 & G12 & G13 & G14 \\ -G12 & 0 & G23 & G24 \\ -G13 & -G23 & 0 & G34 \\ -G14 & -G24 & -G34 & 0 \end{pmatrix}$$

```
{{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

```
(* define inverse of F *)
Finv = Inverse[Fmat];
Finv // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

```
XiConstraints = Table[
   Sum[
     XI[[p]][[i]] Finv[[i]][[j]] Gmat[[j]][[q]] -
      XI[[q]][[i]] Finv[[i]][[j]] Gmat[[j]][[p]],
     {i, 1, 4}, {j, 1, 4}
    ] == 0,
   {p, 1, 4}, {q, 1, 4}
  ];
XiTable = Flatten[FullSimplify[XiConstraints]];
XiTable // MatrixForm
```

```
True
AA (G13 + G24) == 0
EE G12 + 2 BB G13 + DD G14 + CC G23 + AA G34 == 0
CC (G13 + G24) == 0
AA (G13 + G24) == 0
True
DD (G13 + G24) == 0
EE G12 + DD G14 + CC G23 + AA G34 == 2 BB G24
EE G12 + 2 BB G13 + DD G14 + CC G23 + AA G34 == 0
DD (G13 + G24) == 0
True
EE (G13 + G24) == 0
CC (G13 + G24) == 0
EE G12 + DD G14 + CC G23 + AA G34 == 2 BB G24
EE (G13 + G24) == 0
True
```

```
(* remove redundant equations *)
NewConstraints = {};
For[i = 1, i ≤ Length[XiTable], i++,
 OneConstr = XiTable[[i]];
 If[Simplify[OneConstr, NewConstraints] == True,
  (* true, do nothing *)
  Print[ToString[i] <> ": Removing redundant constraint " <> ToString[OneConstr]],
  (* false (?)*)
  Print["??"],
  (* not true/false, that is, a new constraint *)
  NewConstraints = Append[NewConstraints, OneConstr];
  Print[ToString[i] <> ": Adding new constraint " <> ToString[OneConstr]]

 ]
]
```

```
1: Removing redundant constraint True
2: Adding new constraint AA (G13 + G24) == 0
3: Adding new constraint EE G12 + 2 BB G13 + DD G14 + CC G23 + AA G34 == 0
4: Adding new constraint CC (G13 + G24) == 0
5: Removing redundant constraint AA (G13 + G24) == 0
6: Removing redundant constraint True
7: Adding new constraint DD (G13 + G24) == 0
8: Adding new constraint EE G12 + DD G14 + CC G23 + AA G34 == 2 BB G24
9: Removing redundant constraint EE G12 + 2 BB G13 + DD G14 + CC G23 + AA G34 == 0
10: Removing redundant constraint DD (G13 + G24) == 0
11: Removing redundant constraint True
12: Adding new constraint EE (G13 + G24) == 0
13: Removing redundant constraint CC (G13 + G24) == 0
14: Removing redundant constraint EE G12 + DD G14 + CC G23 + AA G34 == 2 BB G24
15: Removing redundant constraint EE (G13 + G24) == 0
16: Removing redundant constraint True
```

```
NewConstraints // MatrixForm
```

$$\begin{pmatrix} AA\,(G13+G24) == 0 \\ EE\,G12 + 2\,BB\,G13 + DD\,G14 + CC\,G23 + AA\,G34 == 0 \\ CC\,(G13+G24) == 0 \\ DD\,(G13+G24) == 0 \\ EE\,G12 + DD\,G14 + CC\,G23 + AA\,G34 == 2\,BB\,G24 \\ EE\,(G13+G24) == 0 \end{pmatrix}$$

- **Claim 3: Writing out F v G neq 0 gives G13 + G24 neq 0:**

```
Simplify[Sum[Signature[{i, j, k, l}] Gmat[[i]][[j]] Fmat[[k]][[l]],
   {i, 1, 4}, {j, 1, 4}, {k, 1, 4}, {l, 1, 4}] == 0]
```

$G13 + G24 == 0$