- **We claim that <w,z>=0 whenever w in W and z in Z.**

```
WAA [a_, b_, i_, j_] :=
 Signature[{a, b}] Signature[{i, j}] w[Min[{a, b}], Max[{a, b}], Min[{i, j}], Max[{i, j}]]

ZAA [a_, b_, i_, j_] :=
 Signature[{a, b}] Signature[{i, j}] z[Min[{a, b}], Max[{a, b}], Min[{i, j}], Max[{i, j}]]
```

Compute <Z,W>

```
InnProd = Sum[
   WAA[p, q, l, m] ZAA[l, m, p, q],
   {p, 1, 4}, {q, 1, 4}, {l, 1, 4}, {m, 1, 4}
  ]
```

4 w[1, 2, 1, 2] z[1, 2, 1, 2] + 4 w[1, 3, 1, 2] z[1, 2, 1, 3] + 4 w[1, 4, 1, 2] z[1, 2, 1, 4] +
 4 w[2, 3, 1, 2] z[1, 2, 2, 3] + 4 w[2, 4, 1, 2] z[1, 2, 2, 4] + 4 w[3, 4, 1, 2] z[1, 2, 3, 4] +
 4 w[1, 2, 1, 3] z[1, 3, 1, 2] + 4 w[1, 3, 1, 3] z[1, 3, 1, 3] + 4 w[1, 4, 1, 3] z[1, 3, 1, 4] +
 4 w[2, 3, 1, 3] z[1, 3, 2, 3] + 4 w[2, 4, 1, 3] z[1, 3, 2, 4] + 4 w[3, 4, 1, 3] z[1, 3, 3, 4] +
 4 w[1, 2, 1, 4] z[1, 4, 1, 2] + 4 w[1, 3, 1, 4] z[1, 4, 1, 3] + 4 w[1, 4, 1, 4] z[1, 4, 1, 4] +
 4 w[2, 3, 1, 4] z[1, 4, 2, 3] + 4 w[2, 4, 1, 4] z[1, 4, 2, 4] + 4 w[3, 4, 1, 4] z[1, 4, 3, 4] +
 4 w[1, 2, 2, 3] z[2, 3, 1, 2] + 4 w[1, 3, 2, 3] z[2, 3, 1, 3] + 4 w[1, 4, 2, 3] z[2, 3, 1, 4] +
 4 w[2, 3, 2, 3] z[2, 3, 2, 3] + 4 w[2, 4, 2, 3] z[2, 3, 2, 4] + 4 w[3, 4, 2, 3] z[2, 3, 3, 4] +
 4 w[1, 2, 2, 4] z[2, 4, 1, 2] + 4 w[1, 3, 2, 4] z[2, 4, 1, 3] + 4 w[1, 4, 2, 4] z[2, 4, 1, 4] +
 4 w[2, 3, 2, 4] z[2, 4, 2, 3] + 4 w[2, 4, 2, 4] z[2, 4, 2, 4] + 4 w[3, 4, 2, 4] z[2, 4, 3, 4] +
 4 w[1, 2, 3, 4] z[3, 4, 1, 2] + 4 w[1, 3, 3, 4] z[3, 4, 1, 3] + 4 w[1, 4, 3, 4] z[3, 4, 1, 4] +
 4 w[2, 3, 3, 4] z[3, 4, 2, 3] + 4 w[2, 4, 3, 4] z[3, 4, 2, 4] + 4 w[3, 4, 3, 4] z[3, 4, 3, 4]

Obtain constraints for z in Z

```
Constraints = {};

For[i = 1, i ≤ 4, i++,
 For[j = 1, j ≤ 4, j++,
  Constraints = Append[Constraints,
    0 ==
     Sum[ ZAA[i, P, j, P] , {P, 1, 4}]
   ]
  ]
 ]
```

Add constraints for w in W

```
For[i = 1, i ≤ 4, i++,
 For[j = i + 1, j ≤ 4, j++,
  For[a = 1, a ≤ 4, a++,
   For[b = a + 1, b ≤ 4, b++,
    Constraints = Append[Constraints,
     Sum[Signature[{i, j, p, q}] WAA[p, q, a, b], {p, 1, 4}, {q, 1, 4}]
      ==
      -Sum[ Signature[{a, b, p, q}] WAA[p, q, i, j] , {p, 1, 4}, {q, 1, 4}]
     ]
    ]
   ]
  ]
 ]
```

```
Constraints // MatrixForm
```

$$
\begin{pmatrix}
0 == z[1, 2, 1, 2] + z[1, 3, 1, 3] + z[1, 4, 1, 4] \\
0 == z[1, 3, 2, 3] + z[1, 4, 2, 4] \\
0 == -z[1, 2, 2, 3] + z[1, 4, 3, 4] \\
0 == -z[1, 2, 2, 4] - z[1, 3, 3, 4] \\
0 == z[2, 3, 1, 3] + z[2, 4, 1, 4] \\
0 == z[1, 2, 1, 2] + z[2, 3, 2, 3] + z[2, 4, 2, 4] \\
0 == z[1, 2, 1, 3] + z[2, 4, 3, 4] \\
0 == z[1, 2, 1, 4] - z[2, 3, 3, 4] \\
0 == -z[2, 3, 1, 2] + z[3, 4, 1, 4] \\
0 == z[1, 3, 1, 2] + z[3, 4, 2, 4] \\
0 == z[1, 3, 1, 3] + z[2, 3, 2, 3] + z[3, 4, 3, 4] \\
0 == z[1, 3, 1, 4] + z[2, 3, 2, 4] \\
0 == -z[2, 4, 1, 2] - z[3, 4, 1, 3] \\
0 == z[1, 4, 1, 2] - z[3, 4, 2, 3] \\
0 == z[1, 4, 1, 3] + z[2, 4, 2, 3] \\
0 == z[1, 4, 1, 4] + z[2, 4, 2, 4] + z[3, 4, 3, 4] \\
2\,w[3, 4, 1, 2] == -2\,w[3, 4, 1, 2] \\
2\,w[3, 4, 1, 3] == 2\,w[2, 4, 1, 2] \\
2\,w[3, 4, 1, 4] == -2\,w[2, 3, 1, 2] \\
2\,w[3, 4, 2, 3] == -2\,w[1, 4, 1, 2] \\
2\,w[3, 4, 2, 4] == 2\,w[1, 3, 1, 2] \\
2\,w[3, 4, 3, 4] == -2\,w[1, 2, 1, 2] \\
-2\,w[2, 4, 1, 2] == -2\,w[3, 4, 1, 3] \\
-2\,w[2, 4, 1, 3] == 2\,w[2, 4, 1, 3] \\
-2\,w[2, 4, 1, 4] == -2\,w[2, 3, 1, 3] \\
-2\,w[2, 4, 2, 3] == -2\,w[1, 4, 1, 3] \\
-2\,w[2, 4, 2, 4] == 2\,w[1, 3, 1, 3] \\
-2\,w[2, 4, 3, 4] == -2\,w[1, 2, 1, 3] \\
2\,w[2, 3, 1, 2] == -2\,w[3, 4, 1, 4] \\
2\,w[2, 3, 1, 3] == 2\,w[2, 4, 1, 4] \\
2\,w[2, 3, 1, 4] == -2\,w[2, 3, 1, 4] \\
2\,w[2, 3, 2, 3] == -2\,w[1, 4, 1, 4] \\
2\,w[2, 3, 2, 4] == 2\,w[1, 3, 1, 4] \\
2\,w[2, 3, 3, 4] == -2\,w[1, 2, 1, 4] \\
2\,w[1, 4, 1, 2] == -2\,w[3, 4, 2, 3] \\
2\,w[1, 4, 1, 3] == 2\,w[2, 4, 2, 3] \\
2\,w[1, 4, 1, 4] == -2\,w[2, 3, 2, 3] \\
2\,w[1, 4, 2, 3] == -2\,w[1, 4, 2, 3] \\
2\,w[1, 4, 2, 4] == 2\,w[1, 3, 2, 3] \\
2\,w[1, 4, 3, 4] == -2\,w[1, 2, 2, 3] \\
-2\,w[1, 3, 1, 2] == -2\,w[3, 4, 2, 4] \\
-2\,w[1, 3, 1, 3] == 2\,w[2, 4, 2, 4] \\
-2\,w[1, 3, 1, 4] == -2\,w[2, 3, 2, 4] \\
-2\,w[1, 3, 2, 3] == -2\,w[1, 4, 2, 4] \\
-2\,w[1, 3, 2, 4] == 2\,w[1, 3, 2, 4] \\
-2\,w[1, 3, 3, 4] == -2\,w[1, 2, 2, 4] \\
2\,w[1, 2, 1, 2] == -2\,w[3, 4, 3, 4] \\
2\,w[1, 2, 1, 3] == 2\,w[2, 4, 3, 4] \\
2\,w[1, 2, 1, 4] == -2\,w[2, 3, 3, 4] \\
2\,w[1, 2, 2, 3] == -2\,w[1, 4, 3, 4] \\
2\,w[1, 2, 2, 4] == 2\,w[1, 3, 3, 4] \\
2\,w[1, 2, 3, 4] == -2\,w[1, 2, 3, 4]
\end{pmatrix}
$$

- **Apply constraints to inner product <Z,W>**

```
Simplify[InnProd, Constraints]
```

0