Claim: perp(U+Z) subset W.

- **We show: If p is a antisymmetric (2,2)-tensor and:**
  <p,u> = 0 for all u in U,
  <p,z> = 0 for all z in Z,
  then p in W.

```
PN[a_, b_, i_, j_] :=
 Signature[{a, b}] Signature[{i, j}] pp[Min[{a, b}], Max[{a, b}], Min[{i, j}], Max[{i, j}]]
```

- **The below constraints are copied from ZisNonDegenerate.nb. If z is in Zperp, then z satisfies the below conditions.**

```
ConsTemp = {z[2, 3, 1, 3] == z[2, 4, 1, 4], z[2, 3, 1, 2] + z[3, 4, 1, 4] == 0,
   z[2, 4, 1, 2] == z[3, 4, 1, 3], z[1, 3, 2, 3] == z[1, 4, 2, 4], z[1, 3, 1, 2] == z[3, 4, 2, 4],
   z[1, 4, 1, 2] + z[3, 4, 2, 3] == 0, z[1, 2, 2, 3] + z[1, 4, 3, 4] == 0,
   z[1, 2, 1, 3] == z[2, 4, 3, 4], z[1, 4, 1, 3] == z[2, 4, 2, 3], z[1, 2, 2, 4] == z[1, 3, 3, 4],
   z[1, 2, 1, 4] + z[2, 3, 3, 4] == 0, z[1, 3, 1, 4] == z[2, 3, 2, 4],
   z[1, 2, 1, 2] + z[1, 4, 1, 4] + z[2, 3, 2, 3] + z[3, 4, 3, 4] == 2 (z[1, 3, 1, 3] + z[2, 4, 2, 4]),
   z[1, 3, 1, 3] + z[2, 4, 2, 4] == z[1, 4, 1, 4] + z[2, 3, 2, 3],
   z[3, 4, 1, 2] == 0, z[2, 4, 1, 3] == 0, z[2, 3, 1, 4] == 0,
   z[1, 4, 2, 3] == 0, z[1, 3, 2, 4] == 0, z[1, 2, 3, 4] == 0};
(* rename tensor from z to pp *)
ConstraintsP = ConsTemp /. {z[var1_, var2_, var3_, var4_] → pp[var1, var2, var3, var4]}
```

```
{pp[2, 3, 1, 3] == pp[2, 4, 1, 4], pp[2, 3, 1, 2] + pp[3, 4, 1, 4] == 0,
 pp[2, 4, 1, 2] == pp[3, 4, 1, 3], pp[1, 3, 2, 3] == pp[1, 4, 2, 4],
 pp[1, 3, 1, 2] == pp[3, 4, 2, 4], pp[1, 4, 1, 2] + pp[3, 4, 2, 3] == 0,
 pp[1, 2, 2, 3] + pp[1, 4, 3, 4] == 0, pp[1, 2, 1, 3] == pp[2, 4, 3, 4],
 pp[1, 4, 1, 3] == pp[2, 4, 2, 3], pp[1, 2, 2, 4] == pp[1, 3, 3, 4],
 pp[1, 2, 1, 4] + pp[2, 3, 3, 4] == 0, pp[1, 3, 1, 4] == pp[2, 3, 2, 4],
 pp[1, 2, 1, 2] + pp[1, 4, 1, 4] + pp[2, 3, 2, 3] + pp[3, 4, 3, 4] ==
  2 (pp[1, 3, 1, 3] + pp[2, 4, 2, 4]), pp[1, 3, 1, 3] + pp[2, 4, 2, 4] ==
  pp[1, 4, 1, 4] + pp[2, 3, 2, 3], pp[3, 4, 1, 2] == 0, pp[2, 4, 1, 3] == 0,
 pp[2, 3, 1, 4] == 0, pp[1, 4, 2, 3] == 0, pp[1, 3, 2, 4] == 0, pp[1, 2, 3, 4] == 0}
```

- **<p,u>=0 for all u in U implies that <p, Id> = 0, so full trace p = 0 when p in perpU.**
  This gives one constraint on p

```
ConstraintsP = Append[ConstraintsP,
   Sum[ PN[p, q, p, q] , {p, 1, 4}, {q, 1, 4}] == 0
  ];
```

ConstraintsP // MatrixForm

```
pp[2, 3, 1, 3] == pp[2, 4, 1, 4]
pp[2, 3, 1, 2] + pp[3, 4, 1, 4] == 0
pp[2, 4, 1, 2] == pp[3, 4, 1, 3]
pp[1, 3, 2, 3] == pp[1, 4, 2, 4]
pp[1, 3, 1, 2] == pp[3, 4, 2, 4]
pp[1, 4, 1, 2] + pp[3, 4, 2, 3] == 0
pp[1, 2, 2, 3] + pp[1, 4, 3, 4] == 0
pp[1, 2, 1, 3] == pp[2, 4, 3, 4]
pp[1, 4, 1, 3] == pp[2, 4, 2, 3]
pp[1, 2, 2, 4] == pp[1, 3, 3, 4]
pp[1, 2, 1, 4] + pp[2, 3, 3, 4] == 0
pp[1, 3, 1, 4] == pp[2, 3, 2, 4]
pp[1, 2, 1, 2] + pp[1, 4, 1, 4] + pp[2, 3, 2, 3] + pp[3, 4, 3, 4] == 2 (pp[1, 3, 1, 3] + pp[2, 4,
pp[1, 3, 1, 3] + pp[2, 4, 2, 4] == pp[1, 4, 1, 4] + pp[2, 3, 2, 3]
pp[3, 4, 1, 2] == 0
pp[2, 4, 1, 3] == 0
pp[2, 3, 1, 4] == 0
pp[1, 4, 2, 3] == 0
pp[1, 3, 2, 4] == 0
pp[1, 2, 3, 4] == 0
2 pp[1, 2, 1, 2] + 2 pp[1, 3, 1, 3] + 2 pp[1, 4, 1, 4] + 2 pp[2, 3, 2, 3] + 2 pp[2, 4, 2, 4] + 2 pp[3, 4
```

- **Next get constraints for an arbitrary tensor to be in W**

```
ConstraintsW = {};
For[i = 1, i ≤ 4, i++,
 For[j = 1, j ≤ 4, j++,
  For[a = 1, a ≤ 4, a++,
   For[b = 1, b ≤ 4, b++,
    NewCon = Simplify[
      Sum[Signature[{i, j, p, q}] PN[p, q, a, b], {p, 1, 4}, {q, 1, 4}]
       ==
       -Sum[ Signature[{a, b, p, q}] PN[p, q, i, j] , {p, 1, 4}, {q, 1, 4}]
      ,
      ConstraintsW];
    If[NewCon == True, , , ConstraintsW = Append[ConstraintsW, NewCon];];
   ]
  ]
 ]
]
ConstraintsW // MatrixForm
```

```
pp[3, 4, 1, 2] == 0
pp[2, 4, 1, 2] == pp[3, 4, 1, 3]
pp[2, 3, 1, 2] + pp[3, 4, 1, 4] == 0
pp[1, 4, 1, 2] + pp[3, 4, 2, 3] == 0
pp[1, 3, 1, 2] == pp[3, 4, 2, 4]
pp[1, 2, 1, 2] + pp[3, 4, 3, 4] == 0
pp[2, 4, 1, 3] == 0
pp[2, 3, 1, 3] == pp[2, 4, 1, 4]
pp[1, 4, 1, 3] == pp[2, 4, 2, 3]
pp[1, 3, 1, 3] + pp[2, 4, 2, 4] == 0
pp[1, 2, 1, 3] == pp[2, 4, 3, 4]
pp[2, 3, 1, 4] == 0
pp[1, 4, 1, 4] + pp[2, 3, 2, 3] == 0
pp[1, 3, 1, 4] == pp[2, 3, 2, 4]
pp[1, 2, 1, 4] + pp[2, 3, 3, 4] == 0
pp[1, 4, 2, 3] == 0
pp[1, 3, 2, 3] == pp[1, 4, 2, 4]
pp[1, 2, 2, 3] + pp[1, 4, 3, 4] == 0
pp[1, 3, 2, 4] == 0
pp[1, 2, 2, 4] == pp[1, 3, 3, 4]
pp[1, 2, 3, 4] == 0
```

- **If a tensor satisfies the above conditions, then the tensor is in W.**

- **Check if these constraints are satisfied; That is, does p belong to W?**

```
For[temp = 1, temp ≤ Length[ConstraintsW], temp++,
 Print[Simplify[ConstraintsW[[temp]], ConstraintsP]];
]
```

```
True
True
True
True
True
True
True
True
True
True
True
True
True
True
True
True
True
True
True
True
True
```