Compute signature of scalar product on Omega^2_2(N)

```
(* parameters
  -a,b (upper) and
  -l,m (lower) parameterize the basis vectors.

  - argP, argQ (upper) and
  - argR, argS (lower) are indices for each basis vector

 The function returns:

    delta_r^[a  delta_s^b]  delta^p_[l  delta^q_m]

*)
BasisForA22[a_, b_, l_, m_, argP_, argQ_, argR_, argS_] :=
 (KroneckerDelta[a, argR] KroneckerDelta[b, argS] -
    KroneckerDelta[b, argR] KroneckerDelta[a, argS])
  (KroneckerDelta[argP, l] KroneckerDelta[argQ, m] -
    KroneckerDelta[argP, m] KroneckerDelta[argQ, l])

ScalarProductOfBasisVectors[a1_, b1_, l1_, m1_, a2_, b2_, l2_, m2_] :=
 Sum[
  BasisForA22[a1, b1, l1, m1, p, q, r, s] BasisForA22[a2, b2, l2, m2, r, s, p, q],
  {p, 1, 4}, {q, 1, 4}, {r, 1, 4}, {s, 1, 4}
 ]

(* By anti-symmetry we may assume that a<b and l<m in the basis vectors *)
```

```
EnumBasis = Table[0, {j, 1, 36}];
tot = 1;
For[i = 1, i ≤ 4, i++,
  For[j = i + 1, j ≤ 4, j++,
    For[a = 1, a ≤ 4, a++,
      For[b = a + 1, b ≤ 4, b++,
        EnumBasis[[tot]] = {i, j, a, b};
        tot = tot + 1;
       ];
     ];
   ];
 ];
EnumBasis // MatrixForm
Length[EnumBasis]
```

1	2	1	2
1	2	1	3
1	2	1	4
1	2	2	3
1	2	2	4
1	2	3	4
1	3	1	2
1	3	1	3
1	3	1	4
1	3	2	3
1	3	2	4
1	3	3	4
1	4	1	2
1	4	1	3
1	4	1	4
1	4	2	3
1	4	2	4
1	4	3	4
2	3	1	2
2	3	1	3
2	3	1	4
2	3	2	3
2	3	2	4
2	3	3	4
2	4	1	2
2	4	1	3
2	4	1	4
2	4	2	3
2	4	2	4
2	4	3	4
3	4	1	2
3	4	1	3
3	4	1	4
3	4	2	3
3	4	2	4
3	4	3	4

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```
MatrixRepresentationOfScalarProduct = Table[
   ScalarProductOfBasisVectors[
    EnumBasis[[i]][[1]],
    EnumBasis[[i]][[2]],
    EnumBasis[[i]][[3]],
    EnumBasis[[i]][[4]],
    EnumBasis[[j]][[1]],
    EnumBasis[[j]][[2]],
    EnumBasis[[j]][[3]],
    EnumBasis[[j]][[4]]
   ]
   ,
   {i, 1, 36},
   {j, 1, 36}
  ];
```

```
MatrixRepresentationOfScalarProduct // MatrixForm
```

4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0
0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0
0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0
0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0
0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4

```
Eigenvalues[MatrixRepresentationOfScalarProduct]
```

```
{-4, -4, -4, -4, -4, -4, -4, -4, -4, -4, -4, -4, -4,
 -4, -4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4}
```

```
% / 4
```

```
{-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1,
 -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

```
(* 15 negative, 21 positive *)

(*Check: trace = sum of eigenvalues*)
Tr[MatrixRepresentationOfScalarProduct]
21 * (+4) + 15 * (-4)

24

24
```