New Perspectives In Teaching Mathematics 2011

Maple-esitys

24.10.2011 Heikki Apiola

Pikainen johdanto

F1 antaa tiiviin taulukon

• Assignments: $Flauseke := (x+y)^5$

$$(x+y)^5 \tag{1.1}$$

expand

$$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 \tag{1.2}$$

factor

$$= (x+y)^5$$

coefficients in y

$$\rightarrow x^5, 5x^4, 10x^3, 10x^2, 5x, 1 \tag{1.3}$$

differentiate w.r.t. x

$$\rightarrow 5(x+y)^4 \tag{1.4}$$

• Functions: $f := x \rightarrow x^2$ evaluate procedure $\rightarrow a^2 = x \rightarrow x^2$ evaluate procedure $\rightarrow x^2$

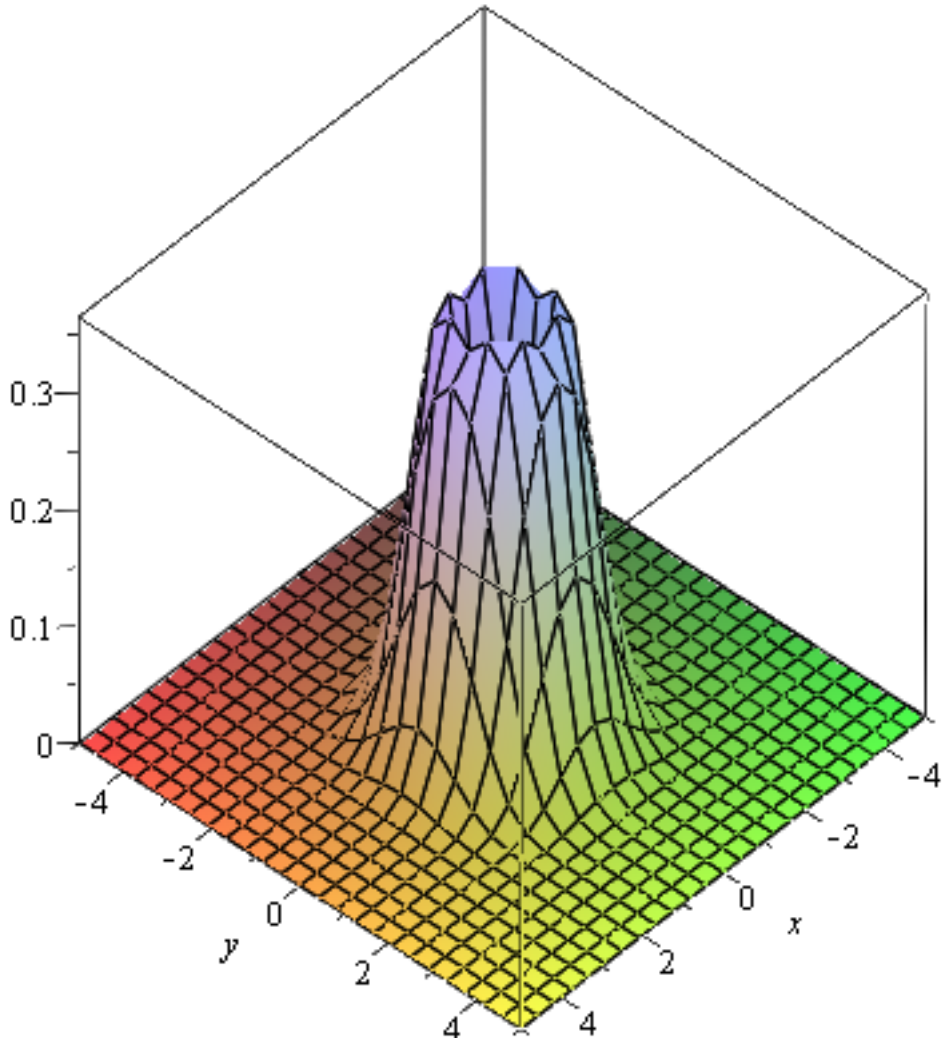
• Equations $a \cdot x^2 + b \cdot x + c = 0$ solve for x $\rightarrow$

$$\left[\left[x = \frac{1}{2}\ \frac{-b+\sqrt{b^2-4ac}}{a}\right], \left[x = -\frac{1}{2}\ \frac{b+\sqrt{b^2-4ac}}{a}\right]\right]$$

• Visualization $(x^2+y^2)\cdot\exp(-x^2-y^2)$

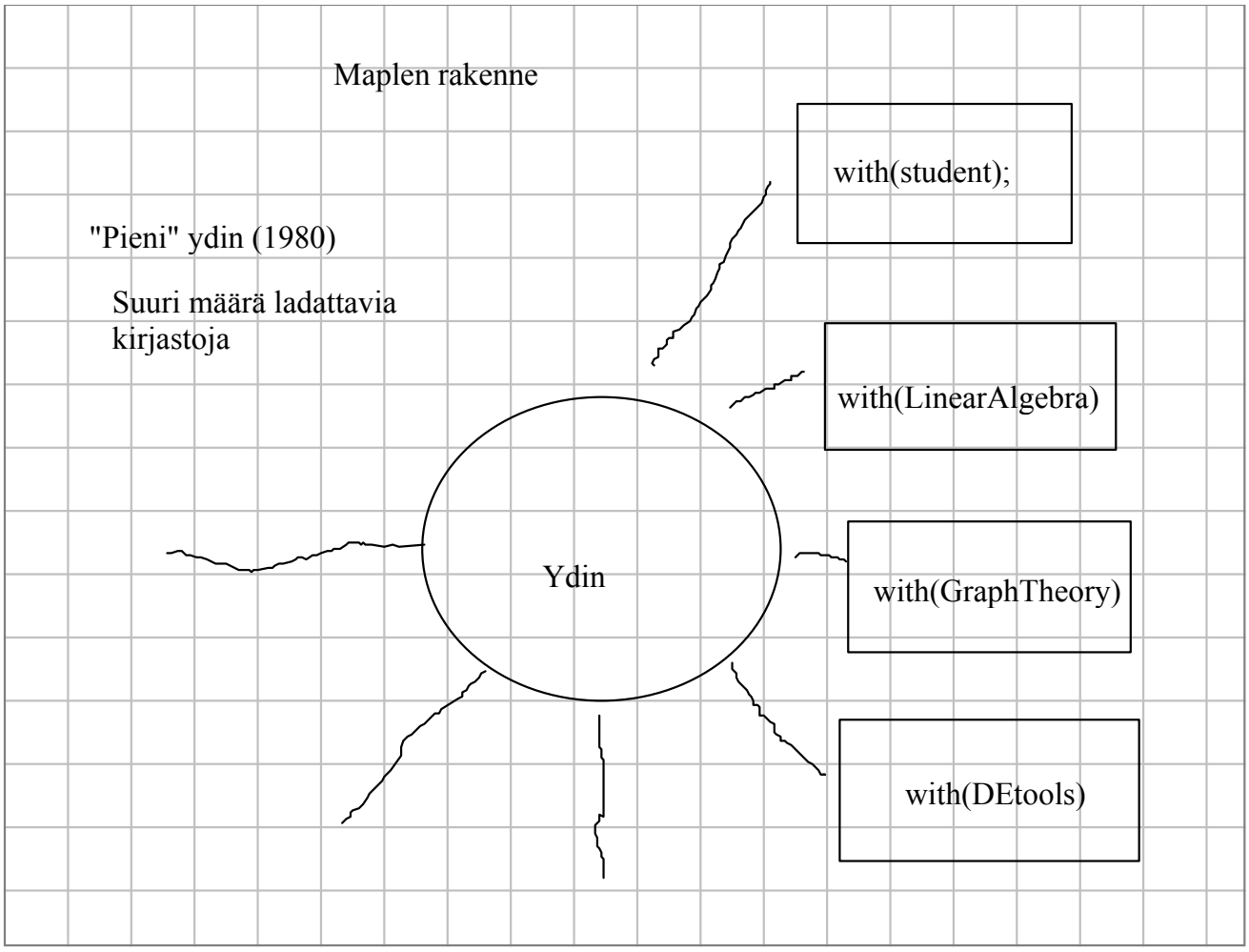
$$(x^2+y^2)\,e^{-x^2-y^2} \tag{1.5}$$

$\rightarrow$



- Tools > Assistants > ODEanalyser

Insert > Canvas



→

? index,packages

Erotusosamäärä ja derivaatta

```
> restart;
```

```
> q:=(f(x0+h)-f(x0))/h;
```

$$q := \frac{f(x0+h)-f(x0)}{h} \tag{2.1}$$

```
> sekantti:=x->f(x0)+q*(x-x0);
```

$$sekantti := x \to f(x0) + q\,(x-x0) \tag{2.2}$$

```
> m:=limit(q,h=0);
```

$$m := \mathrm{D}(f)\,(x0) \tag{2.3}$$

Aika veijari! Maple tiesi!

```
> tangentti:=x->f(x0)+m*(x-x0);
```

$$tangentti := x \to f(x0) + m\,(x-x0) \tag{2.4}$$

```
> h:=1/2;
```

$$h := \frac{1}{2} \tag{2.5}$$

```
> f:=x->1+2*x-x^3;
```

$$f := x \to 1 + 2\,x - x^3 \tag{2.6}$$

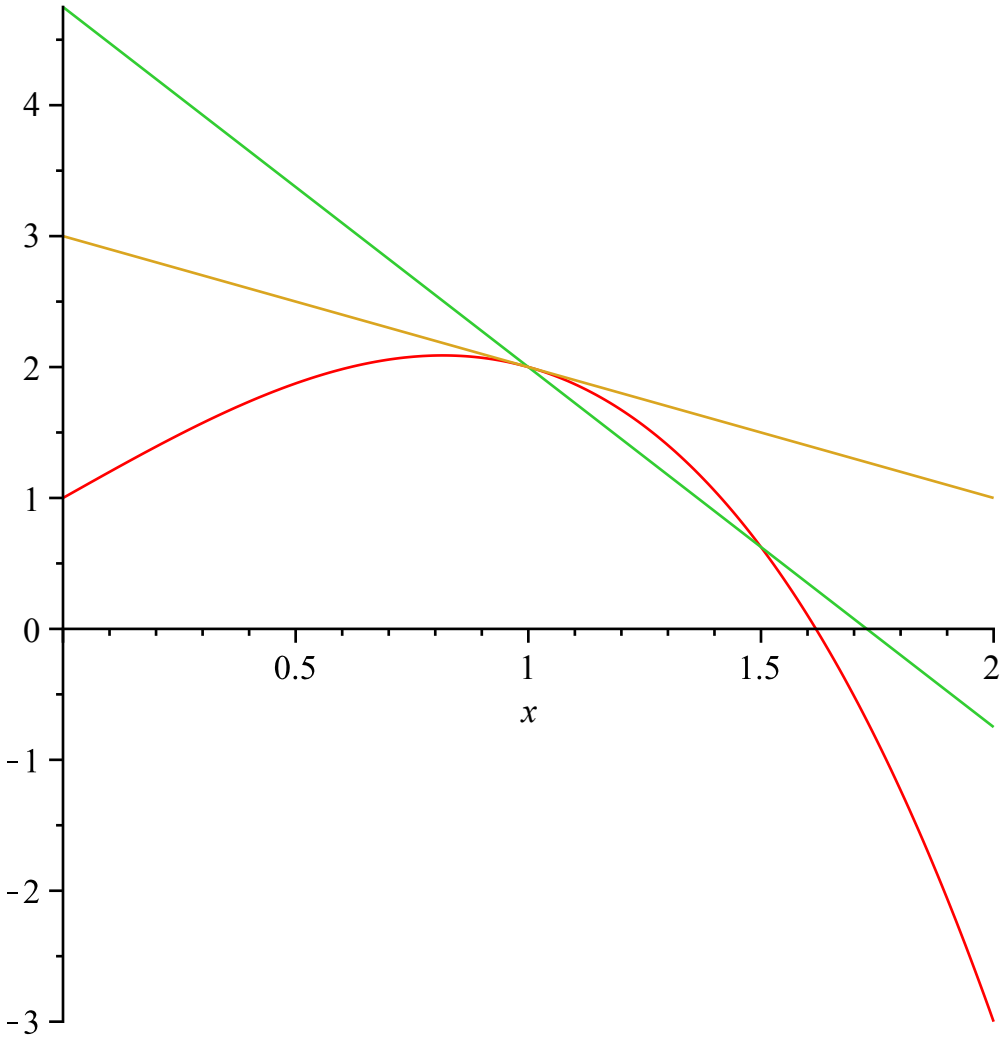
```
> x0:=1;
```

$$x0 := 1 \tag{2.7}$$

```
> m;
```

$$-1 \tag{2.8}$$

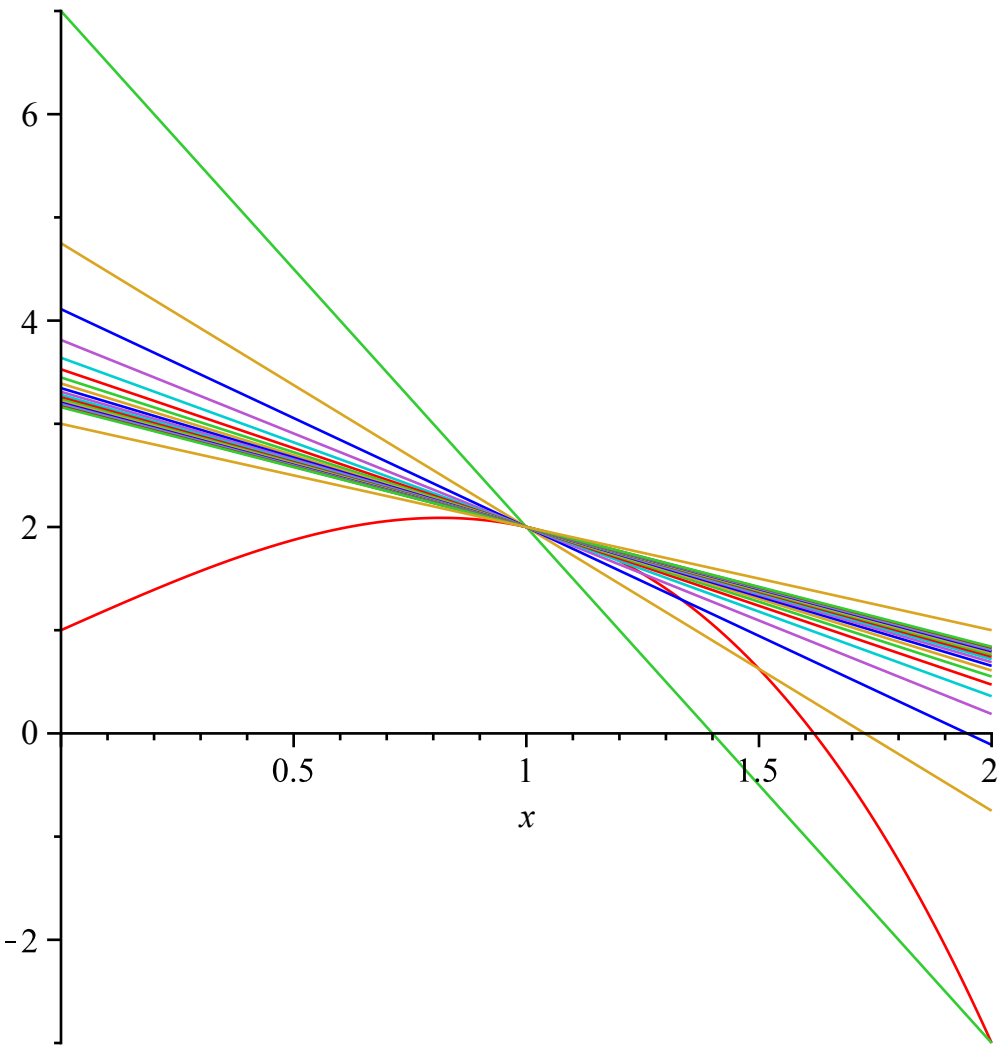
```
> plot([f(x),sekantti(x),tangentti(x)],x=0..2);
```




```
> h:=1/k;
```

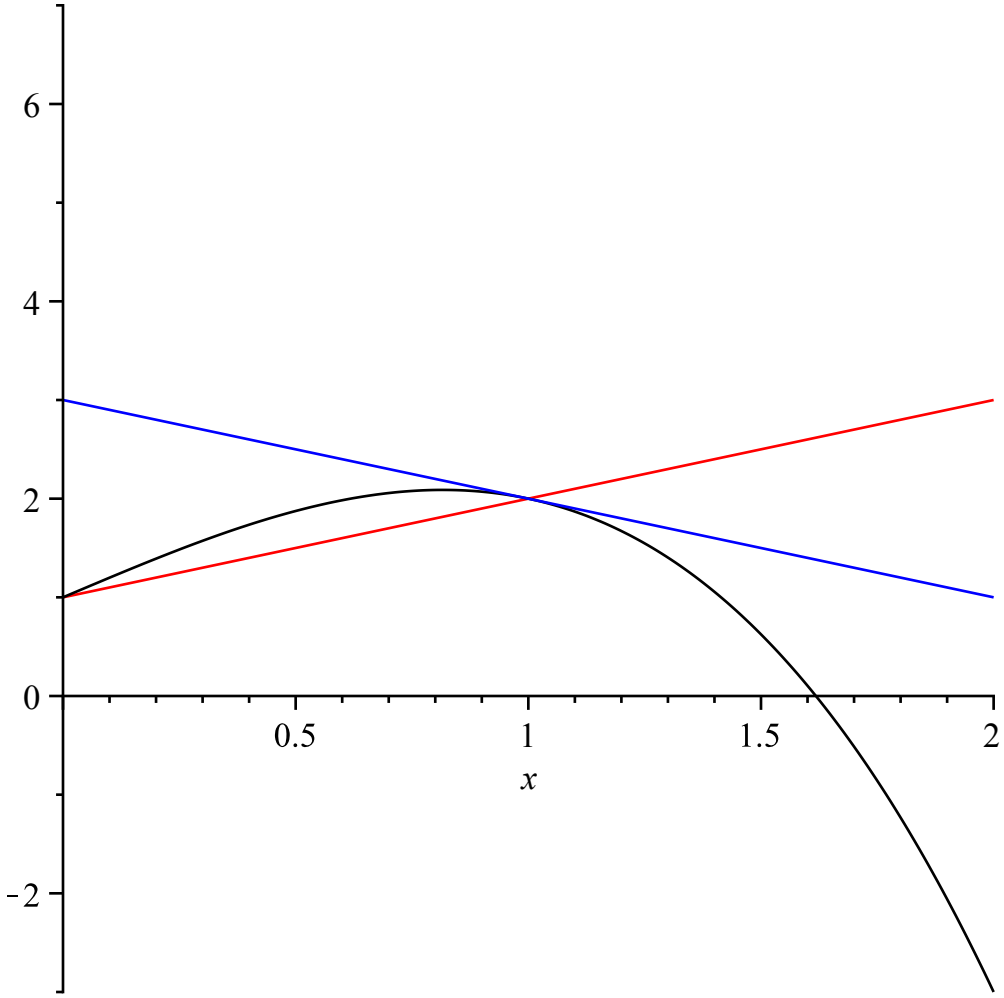
$$h := \frac{1}{k} \tag{2.9}$$

```
> plot([f(x),seq(sekantti(x),k=1..19),tangentti(x)],x=0..2);
```




```
> with(plots):
> h:='h':
> display(plot([f(x),tangentti(x)],x=0..2,color=[black,blue]),
  animate(plot,[sekantti(x),x=0..2],h=-1..1,frames=50));
```

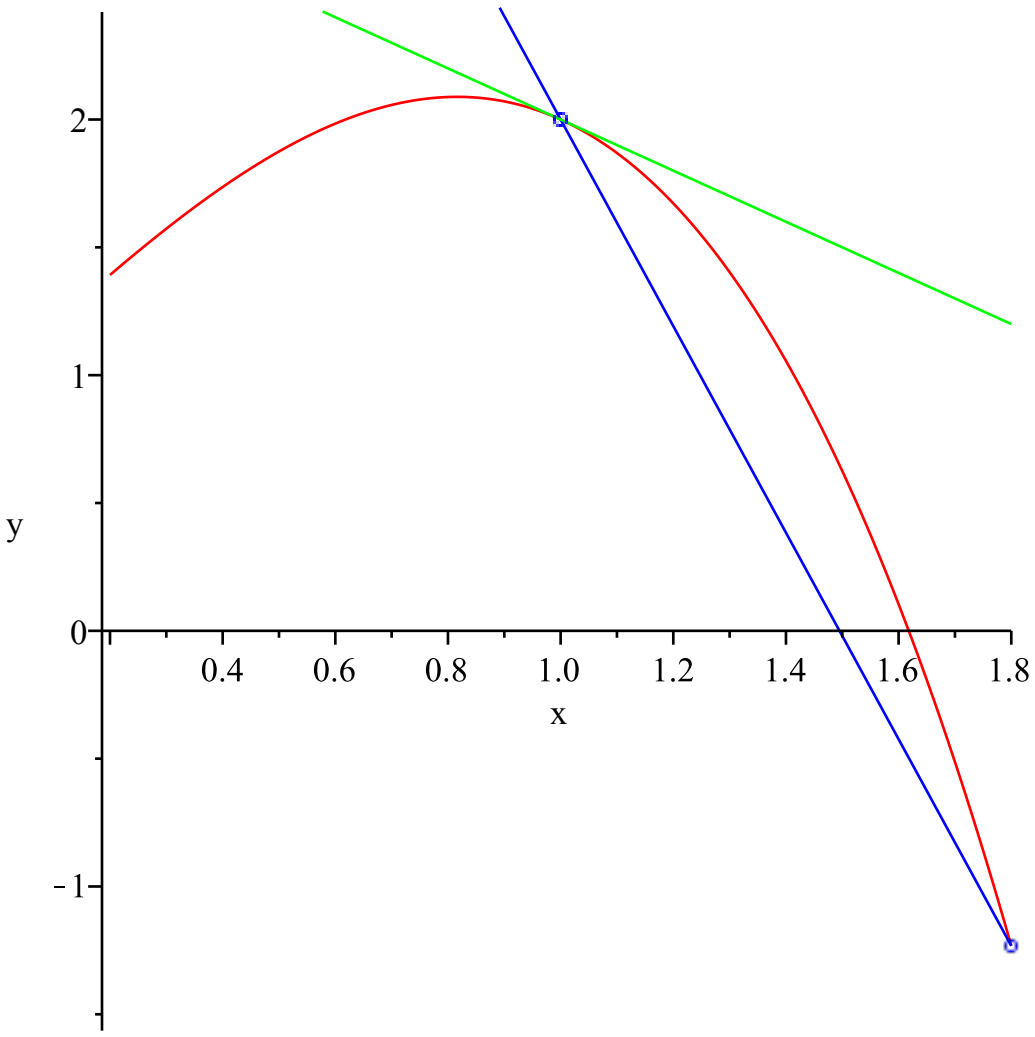
$h = -1.$




```
> ?student
> with(Student[Calculus1]);
```

[*AntiderivativePlot*, *AntiderivativeTutor*, *ApproximateInt*, *ApproximateIntTutor*, *ArcLength*, *ArcLengthTutor*, *Asymptotes*, *Clear*, *CriticalPoints*, *CurveAnalysisTutor*, *DerivativePlot*, *DerivativeTutor*, *DiffTutor*, *ExtremePoints*, *FunctionAverage*, *FunctionAverageTutor*, *FunctionChart*, *FunctionPlot*, *GetMessage*, *GetNumProblems*, *GetProblem*, *Hint*, *InflectionPoints*, *IntTutor*, *Integrand*, *InversePlot*, *InverseTutor*, *LimitTutor*, *MeanValueTheorem*, *MeanValueTheoremTutor*, *NewtonQuotient*, *NewtonsMethod*, *NewtonsMethodTutor*, *PointInterpolation*, *RiemannSum*, *RollesTheorem*, *Roots*, *Rule*, *Show*, *ShowIncomplete*, *ShowSolution*, *ShowSteps*, *Summand*, *SurfaceOfRevolution*, *SurfaceOfRevolutionTutor*, *Tangent*, *TangentSecantTutor*, *TangentTutor*, *TaylorApproximation*, *TaylorApproximationTutor*, *Understand*, *Undo*, *VolumeOfRevolution*, *VolumeOfRevolutionTutor*, *WhatProblem*] **(2.10)**

```
> ?TangentSecantTutor
> TangentSecantTutor(f(x),x=1);
```



Ääriarvotehtävä

Annettu funktio

$$f(x) = \frac{x}{8} + \frac{2}{x}, x > 0.$$

Määritä 3:n levyisen pystykaistaleen minipinta-ala.

Ratkaisu:

Edellä käytettiin "worksheet"-tyyliä, kokeillaan nyt "Document"-tapaa.

```
> restart;  # Tämä worksheet-tyyliin
```

Määritellään Maplelle funktio:

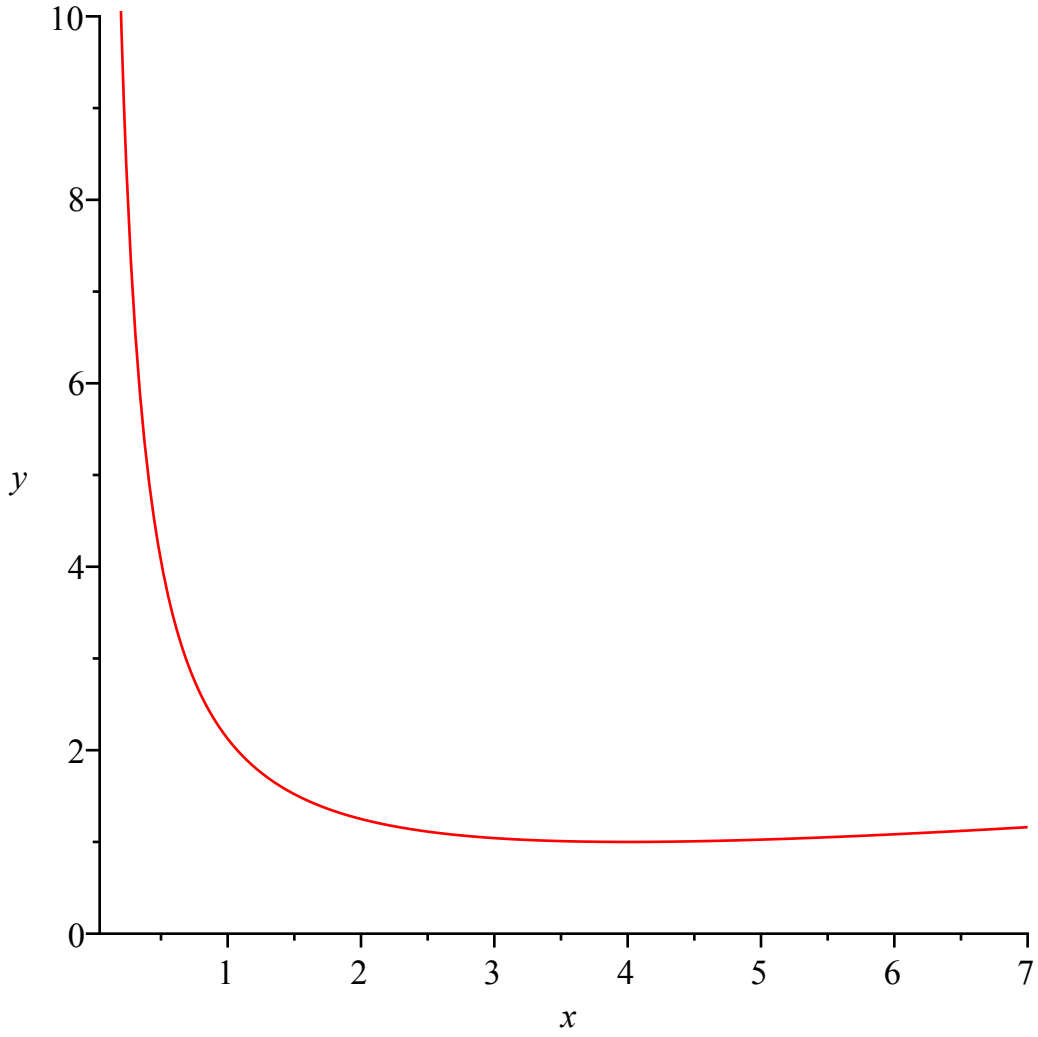
$$f := x \rightarrow \frac{x}{8} + \frac{2}{x}$$

$$x \rightarrow \frac{1}{8}\,x + \frac{2}{x} \qquad \textbf{(3.1)}$$

$$f(1), f(2.4), f(kissa)$$

$$\frac{17}{8},\ 1.133333333,\ \frac{1}{8}\ kissa + \frac{2}{kissa} \qquad \textbf{(3.2)}$$

$plot(f(x), x = 0.1..7, y = 0..10)$



Minimoitava pinta-alafunktio on $F(a) = \int_a^{a+3} f(x)\,dx$, missä a>0.

Määritellään tämä Maplelle funktioksi ja kerrotaan sille, että $a > 0$.

$interface(showassumed = 2)$

$$2 \qquad \textbf{(3.3)}$$

$assume(a > 0)$

a

$$a \qquad \textit{With assumptions on a} \qquad \textbf{(3.4)}$$

$$F := a \to \int_a^{a+3} f(x)\,dx$$

$$a \to \int_a^{a+3} f(x)\,dx \qquad \textbf{(3.5)}$$

$F(1)$

$$\frac{15}{16} + 4\ln(2) \tag{3.6}$$

at 5 digits →

$$3.7101 \tag{3.7}$$

$F(0.5)$

$$4.641820298 \tag{3.8}$$

$\frac{d}{da}F(a)$

$$-\frac{2}{a} + \frac{3}{8} + \frac{2}{a+3}$$

With assumptions on a **(3.9)**

$solve(\mathbf{(3.9)} = 0, a)$

Warning, solve may be ignoring assumptions on the input variables.

$$-\frac{3}{2} + \frac{1}{2}\sqrt{73}, -\frac{3}{2} - \frac{1}{2}\sqrt{73} \tag{3.10}$$

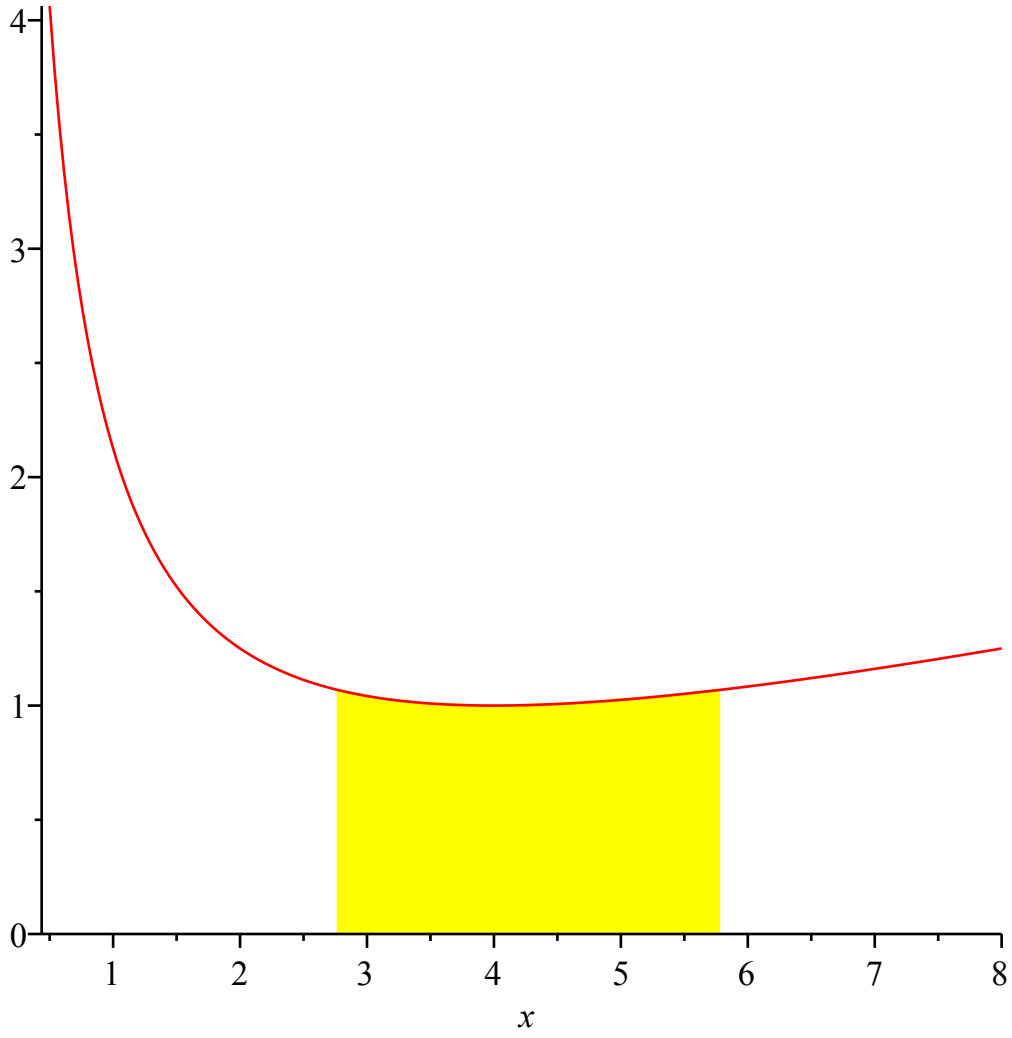
$x_0 := \mathbf{(3.10)}[1]$

$$-\frac{3}{2} + \frac{1}{2}\sqrt{73} \tag{3.11}$$

$evalf(x_0)$

$$2.772001872 \tag{3.12}$$

$with(plots):$
$display(plot(f(x), x = 0.5 .. 8), plot(f(x), x = x_0 .. x_0 + 3, filled = true, color = yellow))$



Esimerkki luentomateriaalista: Määrätty integraali

Riemannin integraali Maplen - työarkilla
Sama pdf-tiedostona (Tämä linkki ei toimi, tiedosto on **integr1.pdf.**)

Syksyn 2011 yo-koe

Ratkaisutyöarkki

Retas tangentes - esimerkki Maplesoftin user-sivulta haetusta

Havainnollistetaan funktion kulkua derivaatan ja tangentin avulla.
Retas tangentes

Ominaisarvoteorian havainnollistus 2x2-matriiseilla

ominaisarvot