Kompleksitason kayraintegraalit

Määritelmä:

```
> int(f(z),z) = int(f(z(t))*diff(z(t),t),t=a..b);
```

$$\int \mathrm{f}(z)\,dz = \int_a^b \mathrm{f}(\mathrm{z}(t))\left(\frac{\partial}{\partial t}\mathrm{z}(t)\right)dt$$

Tassa t->z(t) on ao. polun parametriesitys.
On helppoa maaritella Maple proceduuri, joka implementoi taman:

```
> cint:=proc(f,z,path,t,a,b)
> local dpath,expr;
> dpath:=diff(path,t):
> expr:=simplify(evalc(subs(z=path,f)*dpath)):int(expr,t=a..b);en
  d:
```

Esimerkki: Lasketaan kayraintegraali parabelinkaarta pitkin funktiosta f(z)=z^2 .

```
> polku:=t+I*t^2;
```

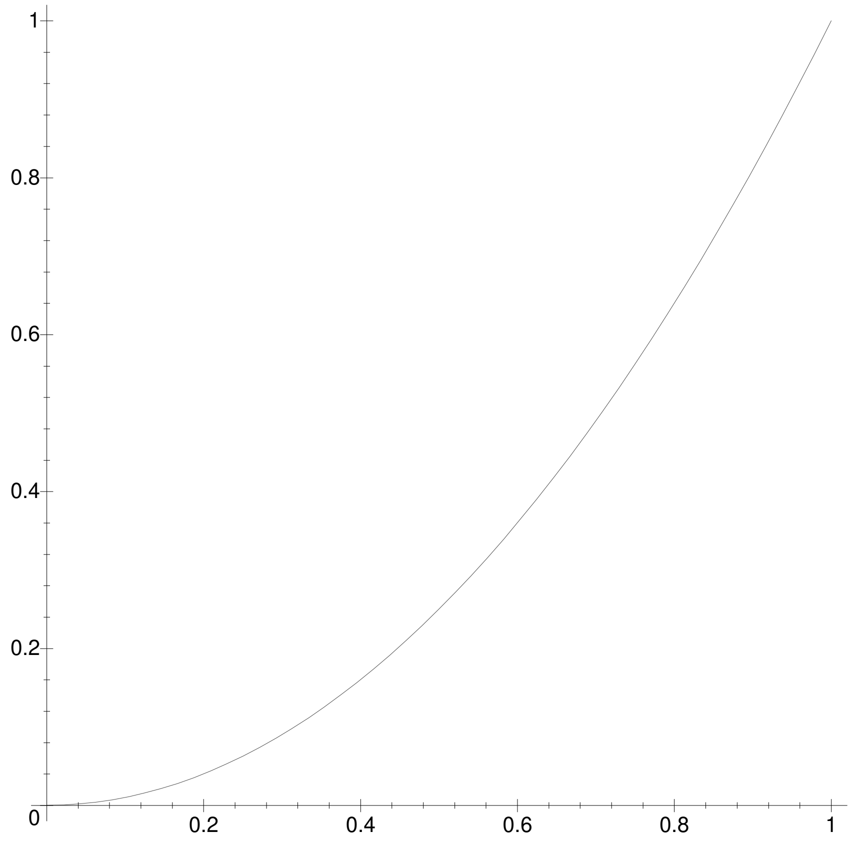
$$polku := t + I\,t^2$$

```
> f:=z^2;
```

$$f := z^2$$

```
> plot([evalc(Re(polku)),evalc(Im(polku)),t=0..1],title=`Integroimis
  polku`);
```

Integroimispolku




```
> cint(f,z,polku,t,0,1);
```

$$-\frac{2}{3}+\frac{2}{3}I$$

Tehdaan viela float-versio (useinhan integrointi ei onnistu suljetussa muodossa).

```
> cintf:=proc(f,z,path,t,a,b)
> local dpath,expr;
> dpath:=diff(path,t):
> expr:=simplify(evalc(subs(z=path,f)*dpath)):evalf(int(expr,t=a..b)
  );end:
>
```

Esimerkkeja: (Harj. 12 loppuviikko, teht. 6)

6 a)

```
> f:=abs(z)^2;
```

$$f:=|z|^2$$

```
> polku:=t*(1+I);
```

$$polku := (1+I)\,t$$

```
> cint(f,z,polku,t,0,1);
```

$$\frac{2}{3} + \frac{2}{3}\,I$$

b)

f::=z+1/z

```
> f:=z+1/z;polku:=exp(I*t);
```

$$f := z + \frac{1}{z}$$

$$polku := \mathbf{e}^{(I\,t)}$$

```
> cint(f,z,polku,t,0,2*Pi);
```

$$2\,I\,\pi$$

```
>
```

c)

```
> f:=1/(z-I);polku:=I+2*exp(I*t);
```

$$f := \frac{1}{z - I}$$

$$polku := I + 2\,\mathbf{e}^{(I\,t)}$$

```
> cint(f,z,polku,t,2*Pi,0);
```

$$-2\,I\,\pi$$

```
>
```