Mat-1.3604 Stationary Processes.

Exercise 13.9. 2007.

1. Let X be a stochastic process defined by

$$X_t = \gamma + \alpha t,$$

where γ is a random variable with Cauchy distribution. Let $\mathcal{D} \subset [0, \infty)$ be a finite or numerable set. Compute the probabilities
a) $\mathbb{P}\left(X_t = 0 \text{ at least for one} t \in \mathcal{D}\right)$.
b) $\mathbb{P}\left(X_t = 0 \text{ at least for one} t \in (1, 2]\right)$.

2. Let X be a stochastic process defined by

$$X_t = Y \cos(t + U),$$

where Y and U are independent random variables, $X \in L^2(\mathbb{P})$, $EX = 0$, and U is uniformly distributed on $[-\pi, \pi]$. Compute the covariance of X.

3. Let X be a stationary process. Compute

$$\operatorname{Var}\left(\int_{-u}^{u} X_s ds\right).$$

4. Let X be a Gaussian process with

$$E\left(X_t\right) = 0 \quad \text{and} \quad E\left(X_t X_{t+\tau}\right) = C(\tau).$$

Find the covariance of the process $\eta_t = \eta_t \eta_{t+s}$; here $t \geq 0$ and s is fixed.

5. Let ξ and θ be independent random variables, θ is uniformly distributed in $[0, 2\pi]$, and ξ has density

$$f_\xi(x) = 2x^3 e^{-\frac{1}{2}x^4} 1_{[0,\infty)}.$$

Show that the process $X_t = \xi^2 \cos\left(2\pi t + \theta\right)$ is Gaussian.

6. Assume that $X = (X_t)_{t \geq 0}$ is stationary Gaussian process. Show that

$$Y_t = \int_t^{t+\tau} X_s ds$$

is also stationary Gaussian.