HUT , Institute of mathematics Gripenberg
Mat-1.196 Mathematics of neural networks
Exercise 4
5.2–13.2.2002

1. Construct, using Kolmogorov's theorem, a neural network that calculates the values of the continuous function $f(x, y)$ when $|x| \leq 1$ and $|y| \leq 1$.

2. Construct, using Kolmogorov's theorem, a neural network with two hidden layers such that the node function is the same at all nodes in each layer, such that the network calculates the values of the continuous function $f(x, y)$ when $|x| \leq 1$ and $|y| \leq 1$.

3. Let B be a symmetric $d \times d$ matrix. Find the point $\mathbf{x}_1$ where the function $f(\mathbf{x}) = \mathbf{x}^{\mathrm{T}} B\mathbf{x}$ ($\mathbf{x}$ is a $d \times 1$ column vector) gets its smallest value on the line $\mathbf{x}_0 + t\mathbf{u}$, $t \in \mathbb{R}$ where $\mathbf{u} \neq \mathbf{0}$. Calculate the answer $\mathbf{x}_1 = (x_1, y_1)$ when $B = \begin{pmatrix} \lambda & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{x}_0 = (x_0, y_0)$ and $\mathbf{u} = f'(\mathbf{x}_0)$. Show that if $|\lambda x_0| = |y_0|$, then $|\lambda x_1| = |y_1|$. What can be said about $\left|\frac{y_1}{y_0}\right|$ in this case.

C1. Write a Matlab-function `fun` that given the arguments such that `fun(x, W, sigma)` calculates the vector $\sigma(\mathbf{w}_i \cdot \mathbf{x} - \tau)$, that is, the threshold is included in the weight-matrix `W`. Give a reasonable name to this function and write it in such a way that if it is given only two arguments, then it assumes some reasonable node function σ.