Mat-5.3741 Theory of Elasticity (5 cp) L **Spring 2007**

Stenberg/Juntunen

Exercise 9

Problem 1

Eliminate the moments M_{ij} and the shear Q_i, $i, j = 1, 2$, in the equations

$$M_{11} = -D\left(\frac{\partial^2 w}{\partial x_1^2} + \nu\frac{\partial^2 w}{\partial x_2^2}\right) \qquad M_{22} = -D\left(\frac{\partial^2 w}{\partial x_2^2} + \nu\frac{\partial^2 w}{\partial x_1^2}\right)$$

$$M_{12} = -(1-\nu)D\frac{\partial^2 w}{\partial x_1 x_2}$$

$$\frac{\partial M_{11}}{\partial x_1} + \frac{\partial M_{12}}{\partial x_2} = Q_1 \qquad \frac{\partial M_{12}}{\partial x_1} + \frac{\partial M_{22}}{\partial x_2} = Q_2$$

$$-\left(\frac{\partial Q_1}{\partial x_1} + \frac{\partial Q_2}{\partial x_2}\right) = f$$

What equation do you get for the deflection w?

Problem 2

Probably the simplest Kirchhoff plate finite element is Morley's nonconforming triangular element. The local space is $P_2(K)$, $K =$ triangle, and the degrees of freedom are the values at the three vertices and the values of the normal derivative at the three midpoints of the edges. Show that this set of degrees of freedom is "unisolvent", i.e. determines the functions uniquely.

Problem 3 **(home exercise)**

Consider a circular domain $\Omega = \{x_1^2 + x_2^2 < R^2\}$ and a uniform load $f = 1/(\pi a^2)$ in the domain $x_1^2 + x_2^2 \leq a^2$. Solve the problem (e.g. with Maple/Mathematica) both for clamped and simply supported case. Print all variables. Study also the limit solution $a \to 0$.

Problem 4 **(home exercise to be handed on Tuesday April 24.)**

Consider a simply supported Kirchhoff plate in the domain $(0, 2a) \times (0, 2b)$. The load is uniform $f = 1/(2cd)$ in the region $(a - c, a + c) \times (b - d, b + d)$. Solve the problem by Fourier series and plot the deflection, the moments, and the shear force. Study also the limit solutions $(c, d) \to (0, 0)$ and $(c, d) \to (a, b)$.