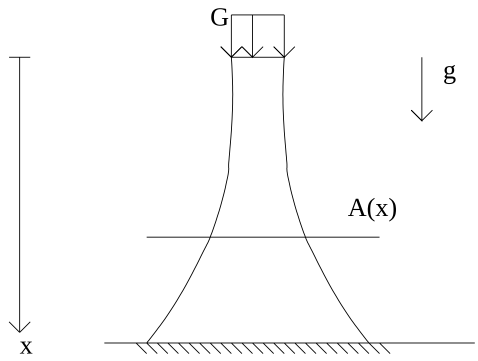
Mat-5.3741 Theory of Elasticity (5 cp) L **Spring 2007**

Stenberg/Juntunen

<u>Exercise 3</u>

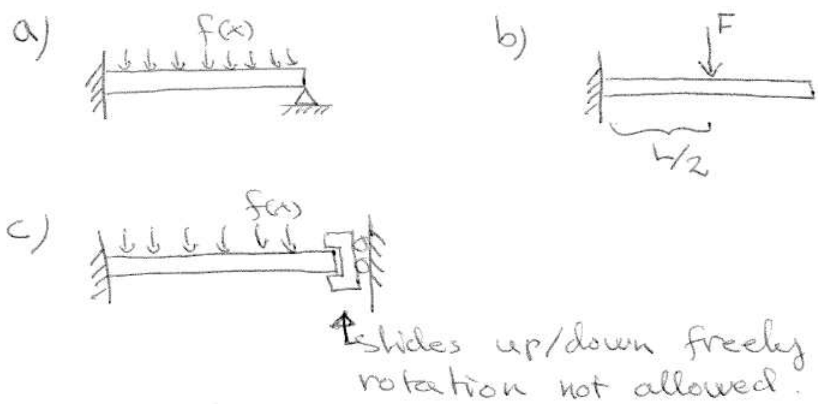
<u>Problem 1</u>

A vertical bar, with varying cross section $A(x)$, is loaded by its own weight (density ρ) and by weight G at the top. What should $A(x)$ be so that stress $\sigma(x)$ will be independent of x?



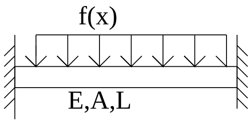
<u>Problem 2</u>

Write the total energy, variational form and boundary value problem for the following beam problems (A, A, L):



Problem 3 **(home exercise)**
Find out the variational forms and boundary value problems for the following energies:
a) $J(v) = \frac{1}{2}\int_0^L EI(v''(x))^2\,dx - Fv(L/2) - Mv'(L/2)$
$K = \{v \,|\, \|v\| < \infty, v(0) = 0\}$
b) $J(v) = \frac{1}{2}\int_0^L EI(v''(x))^2\,dx + k(v(L))^2 - \int_0^L fv\,dx$
$K = \{v \,|\, \|v\| < \infty, v(0) = 0\}$
c) $J(v) = \frac{1}{2}\int_0^L EI(v''(x))^2\,dx + c\int_0^L (v(x))^2\,dx - \int_0^L fv\,dx$
$K = \{v \,|\, \|v\| < \infty\}$.

Problem 4
Consider a beam clamped at both ends.



Find out the Greens function for the solution, i.e. the function $K(x, y)$ such that the solution is $u(x) = \int_0^L K(x, y)f(y)\,dx$.