Mat-1.3379 Dynaamisten systeemien erikoiskurssi **Eirola/Hannukainen**
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Tehtävä 1 on kotitehtävä. Kotitehtävä palautetaan laskuharjoituksiin mennessä huoneen Y323b edessä sijaitsevaan lokeroon tai laskuharjoitusten alussa assistentille.

1. (Kotitehtävä, palautus 9.2)(Problem 5.8 p.71) Show, that for a given $\boldsymbol{A} \in \mathbb{R}^{n\times m}$, the Moore-Penrose pseudoinverse $\boldsymbol{A}^\dagger$ is the solution of minimal Frobenius norm of the minimization problem
$$\min_{\boldsymbol{X}\in\mathbb{R}^{m\times n}} \|\boldsymbol{AX}-\boldsymbol{I}\|_{\mathrm{F}}\,.$$
(So, there exists several solutions $\boldsymbol{X}$, and the one with the minimal norm is the pseudoinverse of $\boldsymbol{A}$.)

2. (Problem 5.12. p.72) Verify the validity of the order of magnitude estimates we gave for the trivial, secant, and tangent predictors (see (4.3), (4.4), (4.5)).

3. (Problem 3.1 p.92) Verify the formulas (from p.92)
$$\begin{aligned} z' &= \beta\alpha + \gamma\alpha z + \delta z^2 + z\boldsymbol{b}^T\boldsymbol{w} + \boldsymbol{w}^T\boldsymbol{B}\boldsymbol{w} + O(|x|^3) =: f(z,\alpha) \\ \boldsymbol{w}' &= \widehat{\boldsymbol{A}}\boldsymbol{w} + \boldsymbol{a}\,z^2 + \dots. \end{aligned}$$
In particular, specify what $\beta,\ \gamma,\ \delta,\ \boldsymbol{a},\ \boldsymbol{b}$ and $\boldsymbol{B}$ are.

4. (Problem 5.11 p.114) Let $\boldsymbol{A} \in \mathbb{R}^{n\times n}$. Provide a formula giving the number of nonzero entries in the ($\frac{1}{2}n(n-1)\times\frac{1}{2}n(n-1)$) matrix $\boldsymbol{A}\odot\boldsymbol{I}$.

5. (Problem 5.12 p.115) Suppose you know two linearly independent vectors in $N(\boldsymbol{f}_{\boldsymbol{u}}(\boldsymbol{u}_0))$. How can you find an approximation of the quadratic form of Theorem 4.2 with 9 evaluations of $\boldsymbol{f}$? Can you do it with less?