Tehtava 1

```
a={{1,1},{3,1}}
```

```
{{1, 1}, {3, 1}}
```

Kokeillaan aluksi pistetta {-0.1,0.1}

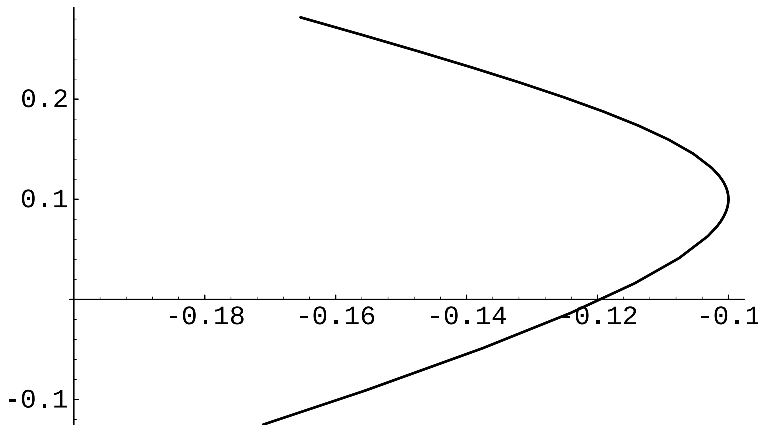
```
vas=NDSolve[{x1'[t]==x1[t]+x2[t],x2'[t]==3 x1[t]+x2[t]
,x1[0]==-0.1,x2[0]==0.1},{x1[t],x2[t]},{t,-1,1}]
```

```
{{x1[t] -> InterpolatingFunction[{-1., 1.}, <>][t],
   x2[t] -> InterpolatingFunction[{-1., 1.}, <>][t]}}
```

```
vas={x1[t],x2[t]}/.vas[[1]]
```

```
{InterpolatingFunction[{-1., 1.}, <>][t],
  InterpolatingFunction[{-1., 1.}, <>][t]}
```

```
ParametricPlot[vas,{t,-1,1}]
```




```
-Graphics-
```

Ja sitten pistetta {0.1,0.1}

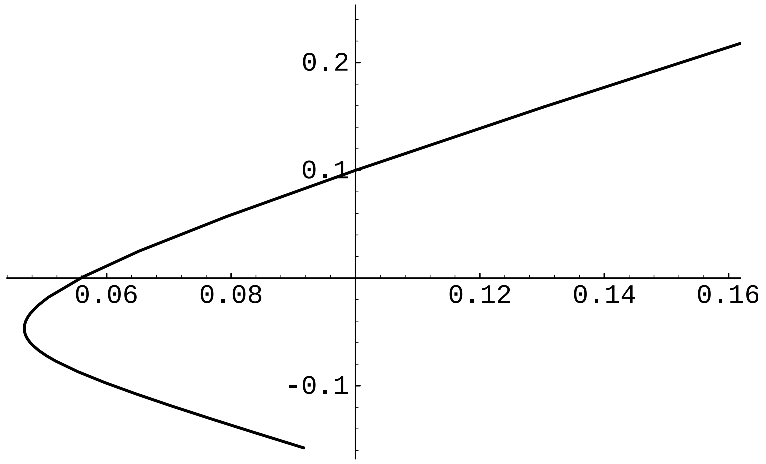
```
vas=NDSolve[{x1'[t]==x1[t]+x2[t],x2'[t]==3 x1[t]+x2[t]
,x1[0]==0.1,x2[0]==0.1},{x1[t],x2[t]},{t,-2,1}]
```

```
{{x1[t] -> InterpolatingFunction[{-2., 1.}, <>][t],
   x2[t] -> InterpolatingFunction[{-2., 1.}, <>][t]}}
```

```
vas={x1[t],x2[t]}/.vas[[1]]
```

```
{InterpolatingFunction[{-2., 1.}, <>][t],
  InterpolatingFunction[{-2., 1.}, <>][t]}
```

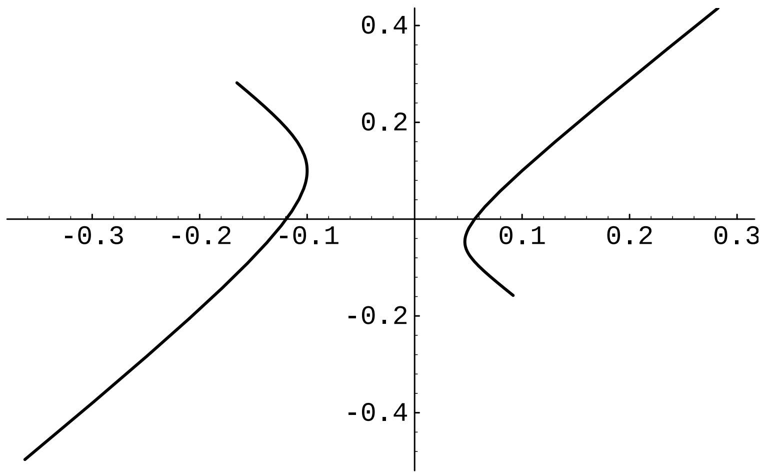
ParametricPlot[vas,{t,-2,1}]



-Graphics-

Yhteiskuva on siis taman nakoinen (vaikuttaisi satulalta)

Show[%41,%48]



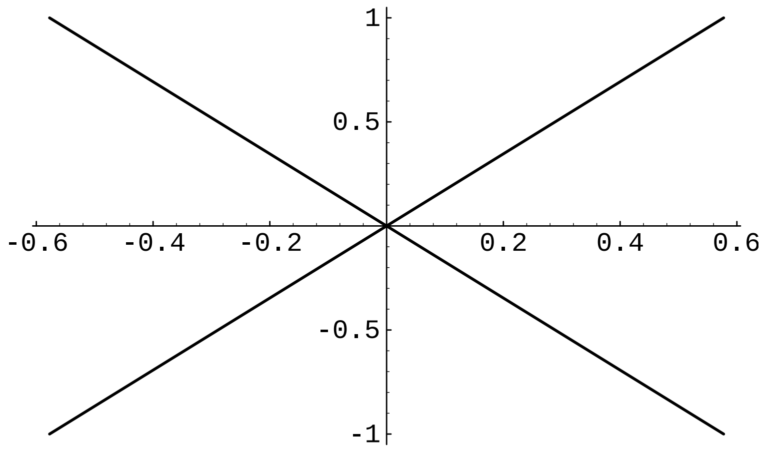
-Graphics-

Eigenvectors[a]

$$\{\{-(\frac{1}{\mathrm{Sqrt}[3]}), 1\}, \{\frac{1}{\mathrm{Sqrt}[3]}, 1\}\}$$

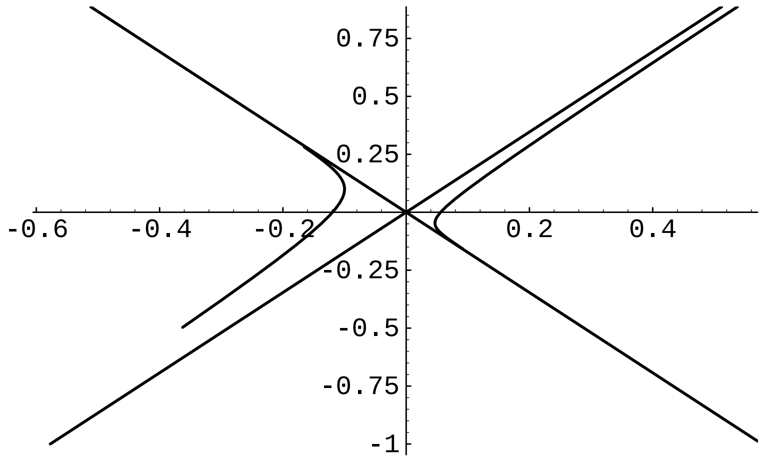
Piirretaan viela ominaisvektoreihin liittyvat suorat

```
ParametricPlot[{%50[[1]] t,%50[[2]] t},{t,-1,1}]
```



Asetetaan kaikki samaan kuvaan

```
Show[%49,%51]
```




```
-Graphics-
```

```
N[Eigenvalues[a]]
```

```
{-0.732051, 2.73205}
```

Origo on tosiaankin siis satula.

Tehtava 2

```
f1=2 x1 - 2 x1 x2
```

```
2 x1 - 2 x1 x2
```

```
f2=2 x2 - x1^2 + x2^2
```

$-x1^2 + 2\ x2 + x2^2$

```
tppisteet=Solve[{f1,f2}=={0,0},{x1,x2}]
```

```
{{x1 -> 0, x2 -> -2}, {x1 -> 0, x2 -> 0},
  {x1 -> -Sqrt[3], x2 -> 1}, {x1 -> Sqrt[3], x2 -> 1}}
```

```
jacobi=Outer[D,{f1,f2},{x1,x2}]
```

```
{{2 - 2 x2, -2 x1}, {-2 x1, 2 + 2 x2}}
```

```
jacobi /. tppisteet
```

```
{{{6, 0}, {0, -2}}, {{2, 0}, {0, 2}},
  {{0, 2 Sqrt[3]}, {2 Sqrt[3], 4}},
  {{0, -2 Sqrt[3]}, {-2 Sqrt[3], 4}}}
```

```
Table[Eigenvalues[%62[[i]]],{i,1,4}]
```

```
{{-2, 6}, {2, 2}, {-2, 6}, {-2, 6}}
```

Lineaarisaation mukaan siis toinen tasapainopiste on lahde, muut satuloita.

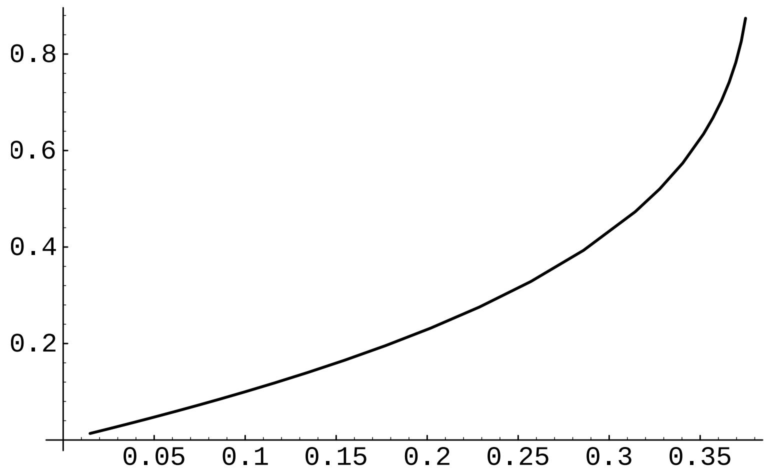
```
vas=NDSolve[{x1'[t]==2 x1[t]-2 x1[t] x2[t],
x2'[t]==2 x2[t] - x1[t]^2 + x2[t]^2, x1[0]==0.1,x2[0]==0.1},
{x1[t],x2[t]},{t,-1,1}]
```

```
{{x1[t] -> InterpolatingFunction[{-1., 1.}, <>][t],
   x2[t] -> InterpolatingFunction[{-1., 1.}, <>][t]}}
```

```
vas={x1[t],x2[t]}/.vas[[1]]
```

```
{InterpolatingFunction[{-1., 1.}, <>][t],
  InterpolatingFunction[{-1., 1.}, <>][t]}
```

```
ParametricPlot[vas,{t,-1,1}]
```




```
-Graphics-
```

Origo olikin lahde, joten ratkaisujen tulisi paeta sielta.

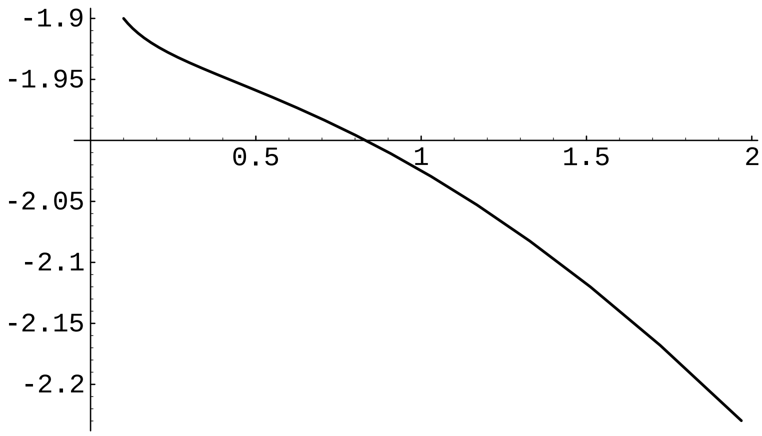
```
vas=NDSolve[{x1'[t]==2 x1[t]-2 x1[t] x2[t],
x2'[t]==2 x2[t] - x1[t]^2 + x2[t]^2, x1[0]==0.1,x2[0]==-1.9},
{x1[t],x2[t]},{t,0,0.5}]
```

```
{{x1[t] -> InterpolatingFunction[{0., 0.5}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 0.5}, <>][t]}}
```

```
vas={x1[t],x2[t]}/.vas[[1]]
```

```
{InterpolatingFunction[{0., 0.5}, <>][t],
  InterpolatingFunction[{0., 0.5}, <>][t]}
```

```
ParametricPlot[vas,{t,0,0.5}]
```




```
-Graphics-
```

Kaytos siis kuten satulassa.

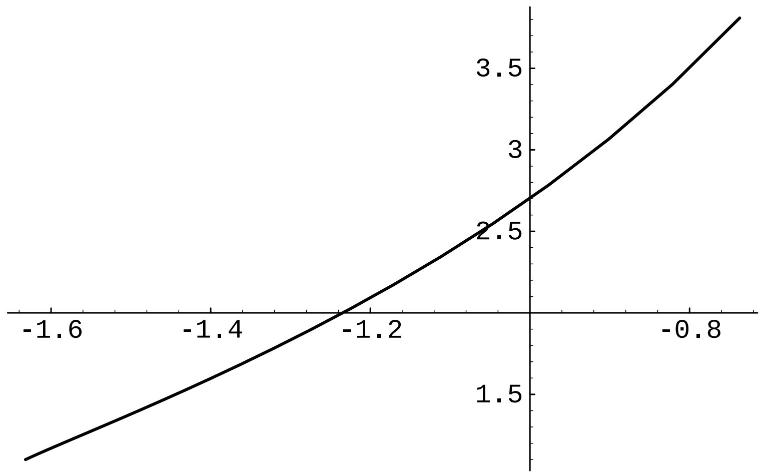
```
vas=NDSolve[{x1'[t]==2 x1[t]-2 x1[t] x2[t],
x2'[t]==2 x2[t] - x1[t]^2 + x2[t]^2, x1[0]==-Sqrt[3]+0.1,
x2[0]==1.1},{x1[t],x2[t]},{t,0,0.5}]
```

```
{{x1[t] -> InterpolatingFunction[{0., 0.5}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 0.5}, <>][t]}}
```

```
vas={x1[t],x2[t]}/.vas[[1]]
```

```
{InterpolatingFunction[{0., 0.5}, <>][t],
  InterpolatingFunction[{0., 0.5}, <>][t]}
```

```
ParametricPlot[vas,{t,0,0.5}]
```




```
-Graphics-
```

Satulaa...

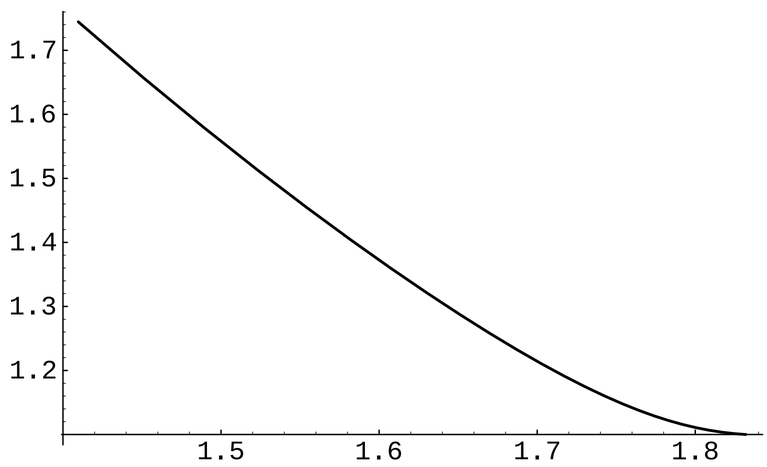
```
vas=NDSolve[{x1'[t]==2 x1[t]-2 x1[t] x2[t],
x2'[t]==2 x2[t] - x1[t]^2 + x2[t]^2, x1[0]==Sqrt[3]+0.1,
x2[0]==1.1},{x1[t],x2[t]},{t,0,0.5}]

{{x1[t] -> InterpolatingFunction[{0., 0.5}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 0.5}, <>][t]}}

vas={x1[t],x2[t]}/.vas[[1]]

{InterpolatingFunction[{0., 0.5}, <>][t],
  InterpolatingFunction[{0., 0.5}, <>][t]}

ParametricPlot[vas,{t,0,0.5}]
```




```
-Graphics-
```

Ja jalleen satula.

```
{InterpolatingFunction[{0., 0.5}, <>][t],
  InterpolatingFunction[{0., 0.5}, <>][t]}
```

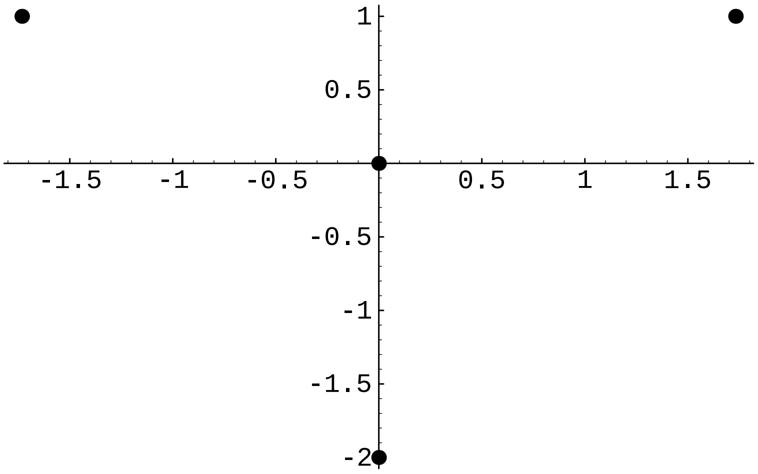
tppisteet

```
{{x1 -> 0, x2 -> -2}, {x1 -> 0, x2 -> 0},
  {x1 -> -Sqrt[3], x2 -> 1}, {x1 -> Sqrt[3], x2 -> 1}}
```

{x1,x2} /. tppisteet

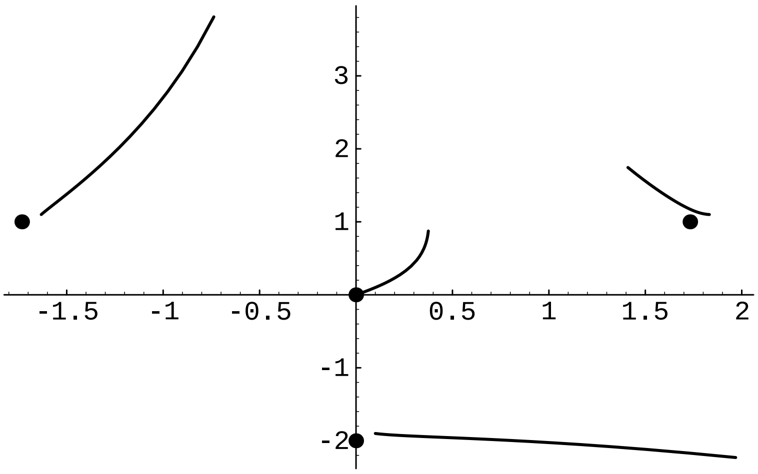
```
{{0, -2}, {0, 0}, {-Sqrt[3], 1}, {Sqrt[3], 1}}
```

ListPlot[%130,Prolog->PointSize[0.02]]




```
-Graphics-
```

Show[%111,%118,%122,%128,%133,Prolog->PointSize[0.02]]




```
-Graphics-
```

Tehtava 3

h=x2^2 / 2 + x1^2 /2 - x1^4 / 4

$$\frac{x1^2}{2} - \frac{x1^4}{4} + \frac{x2^2}{2}$$

htppisteet=Solve[{D[h,x1],D[h,x2]}=={0,0},{x1,x2}]

```
{{x1 -> -1, x2 -> 0}, {x1 -> 0, x2 -> 0}, {x1 -> 1, x2 -> 0}}
```

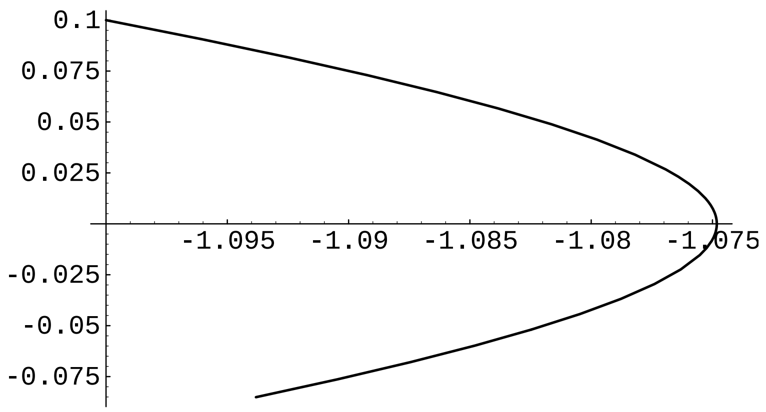
vas=NDSolve[{x1'[t]==x2[t], x2'[t]==-x1[t]+x1[t]^3,
x1[0]==-1.1,x2[0]==0.1},{x1[t],x2[t]},{t,0,1}]

```
{{x1[t] -> InterpolatingFunction[{0., 1.}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 1.}, <>][t]}}
```

vas={x1[t],x2[t]} /. vas[[1]]

```
{InterpolatingFunction[{0., 1.}, <>][t],
  InterpolatingFunction[{0., 1.}, <>][t]}
```

ParametricPlot[vas,{t,0,1}]



-Graphics-

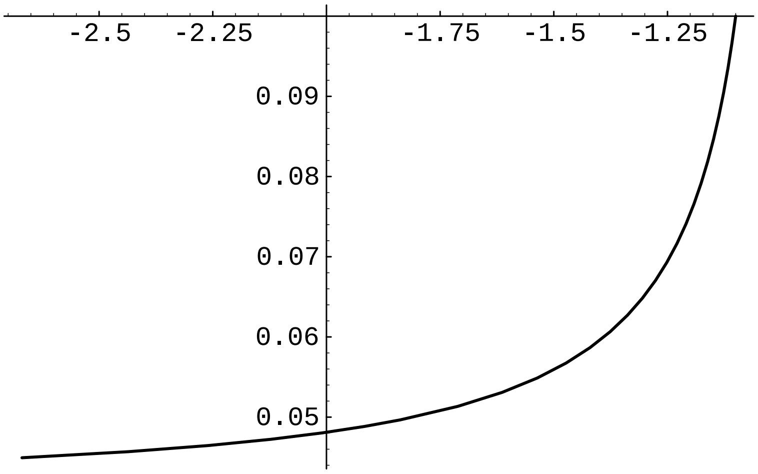
vas=NDSolve[{x2'[t]==-x2[t], x1'[t]==-x1[t]+x1[t]^3,
x1[0]==-1.1,x2[0]==0.1},{x1[t],x2[t]},{t,0,0.8}]

```
{{x1[t] -> InterpolatingFunction[{0., 0.8}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 0.8}, <>][t]}}
```

vas={x1[t],x2[t]} /. vas[[1]]

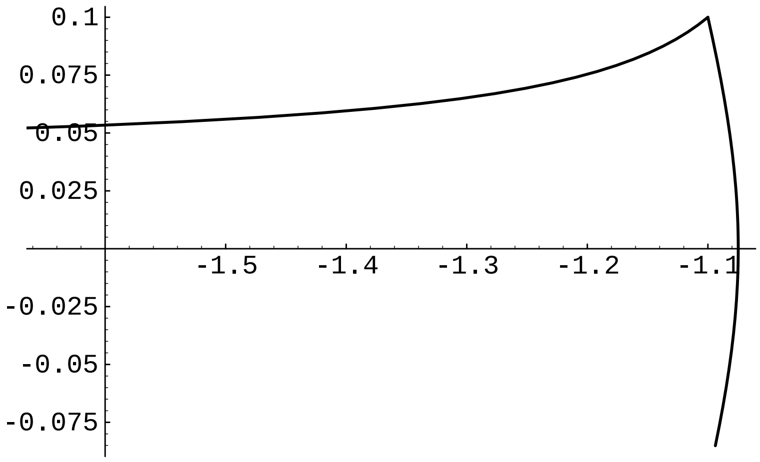
```
{InterpolatingFunction[{0., 0.8}, <>][t],
  InterpolatingFunction[{0., 0.8}, <>][t]}
```

ParametricPlot[vas,{t,0,0.8}]




```
-Graphics-
```

Show[%5,%10]




```
-Graphics-
```

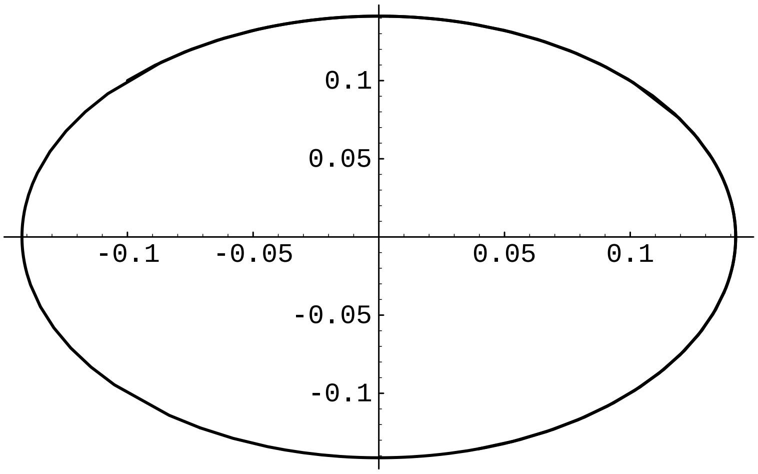
vas=NDSolve[{x1'[t]==x2[t], x2'[t]==-x1[t]+x1[t]^3,
x1[0]==-0.1,x2[0]==0.1},{x1[t],x2[t]},{t,0,10}]

```
{{x1[t] -> InterpolatingFunction[{0., 10.}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 10.}, <>][t]}}
```

vas={x1[t],x2[t]} /. vas[[1]]

```
{InterpolatingFunction[{0., 10.}, <>][t],
  InterpolatingFunction[{0., 10.}, <>][t]}
```

```
ParametricPlot[vas,{t,0,10}]
```




```
-Graphics-
```

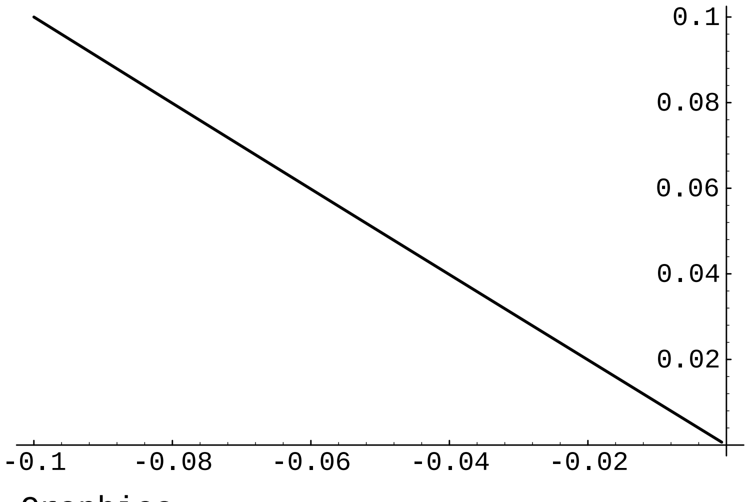
```
vas=NDSolve[{x2'[t]==-x2[t], x1'[t]==-x1[t]+x1[t]^3,
x1[0]==-0.1,x2[0]==0.1},{x1[t],x2[t]},{t,0,5}]
```

```
{{x1[t] -> InterpolatingFunction[{0., 5.}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 5.}, <>][t]}}
```

```
vas={x1[t],x2[t]} /. vas[[1]]
```

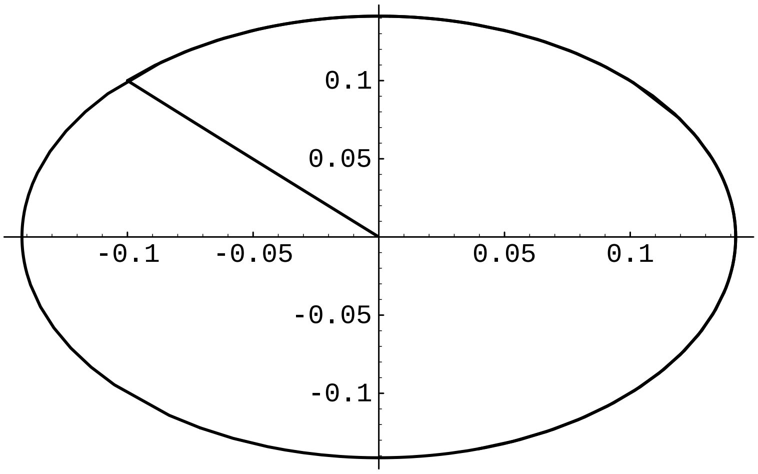
```
{InterpolatingFunction[{0., 5.}, <>][t],
  InterpolatingFunction[{0., 5.}, <>][t]}
```

```
ParametricPlot[vas,{t,0,5}]
```




```
-Graphics-
```

```
Show[%27,%21]
```




```
-Graphics-
```

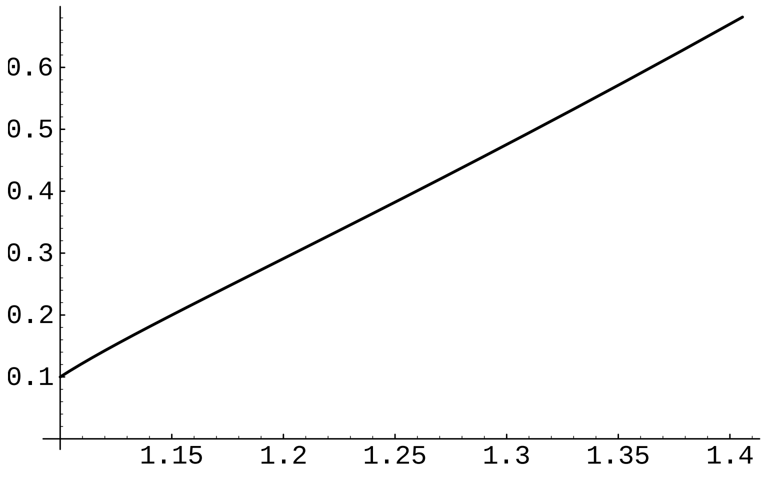
```
vas=NDSolve[{x1'[t]==x2[t], x2'[t]==-x1[t]+x1[t]^3,
x1[0]==1.1,x2[0]==0.1},{x1[t],x2[t]},{t,0,1}]
```

```
{{x1[t] -> InterpolatingFunction[{0., 1.}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 1.}, <>][t]}}
```

```
vas={x1[t],x2[t]} /. vas[[1]]
```

```
{InterpolatingFunction[{0., 1.}, <>][t],
  InterpolatingFunction[{0., 1.}, <>][t]}
```

```
ParametricPlot[vas,{t,0,1}]
```




```
-Graphics-
```

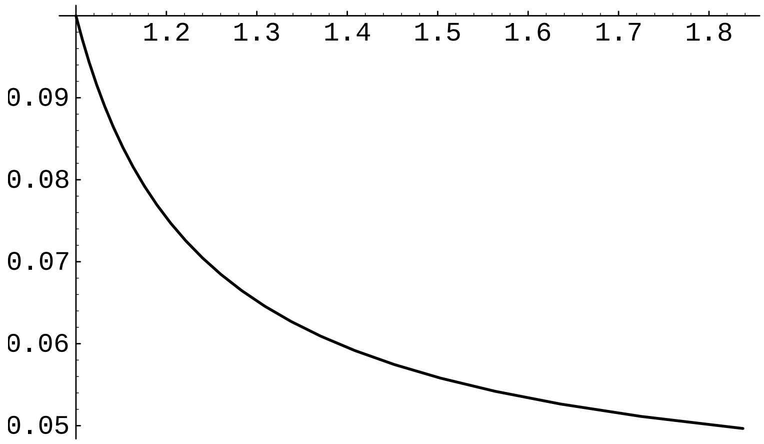
```
vas=NDSolve[{x2'[t]==-x2[t], x1'[t]==-x1[t]+x1[t]^3,
x1[0]==1.1,x2[0]==0.1},{x1[t],x2[t]},{t,0,0.7}]
```

```
{{x1[t] -> InterpolatingFunction[{0., 0.7}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 0.7}, <>][t]}}
```

```
vas={x1[t],x2[t]} /. vas[[1]]
```

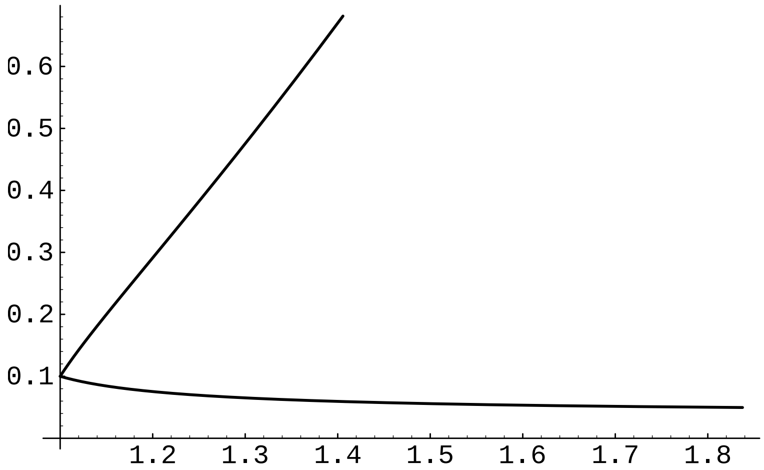
```
{InterpolatingFunction[{0., 0.7}, <>][t],
  InterpolatingFunction[{0., 0.7}, <>][t]}
```

```
ParametricPlot[vas,{t,0,0.7}]
```




```
-Graphics-
```

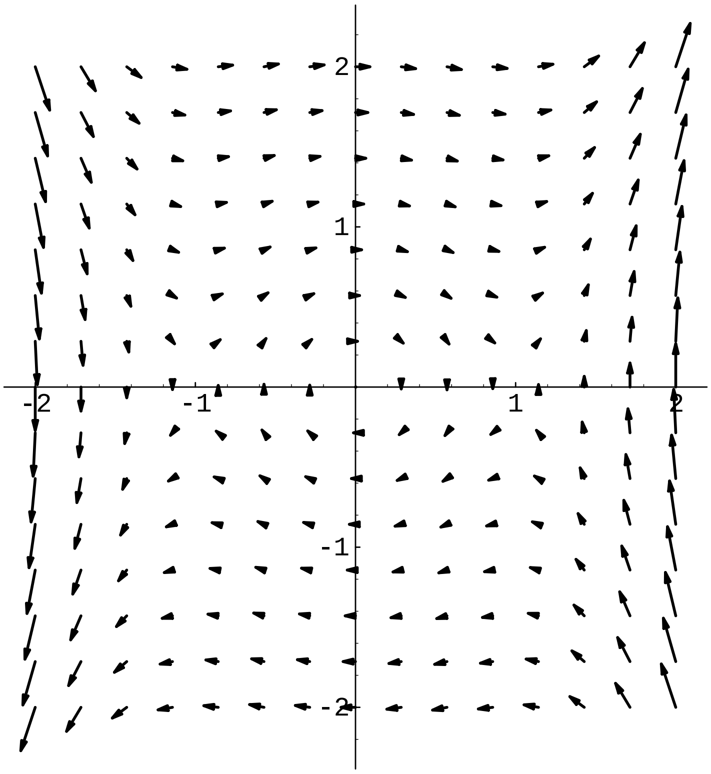
```
Show[%36,%40]
```




```
-Graphics-
```

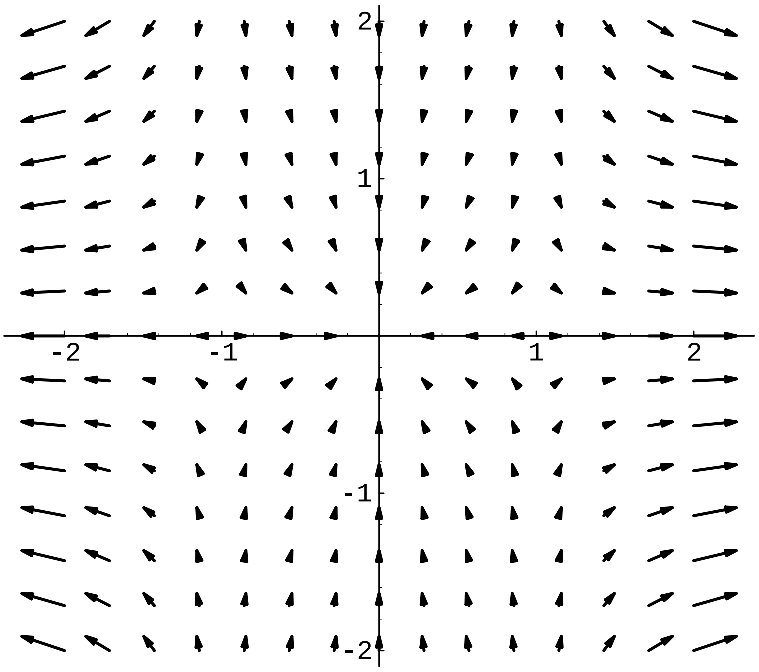
```
<<Graphics‘PlotField‘
```

```
PlotVectorField[{x2,-x1+x1^3},{x1,-2,2},{x2,-2,2},Axes->True]
```




```
-Graphics-
```

```
PlotVectorField[{-x1+x1^3,-x2},{x1,-2,2},{x2,-2,2},Axes->True]
```




```
-Graphics-
```

Tasta naemme, etta vastaavat parit ovat satula <-> satula, keskus <-> nielu.

Tehtava 4

```
f={-x2+x1 (mu - x1^2 - x2^2),x1 + x2 (mu-x1^2-x2^2)}
```

$\{-x2 + x1 (mu - x1^2 - x2^2), x1 + x2 (mu - x1^2 - x2^2)\}$

```
Outer[D,f,{x1,x2}] /. {x1->0, x2->0}
```

```
{{mu, -1}, {1, mu}}
```

```
Eigenvalues[%53]
```

```
{-I + mu, I + mu}
```

Siis selvasti mu on ominaisarvojen reaaliosa eli se vaikuttaa oleellisesti ratkaisun kayttaytymiseen origon ymparistossa.

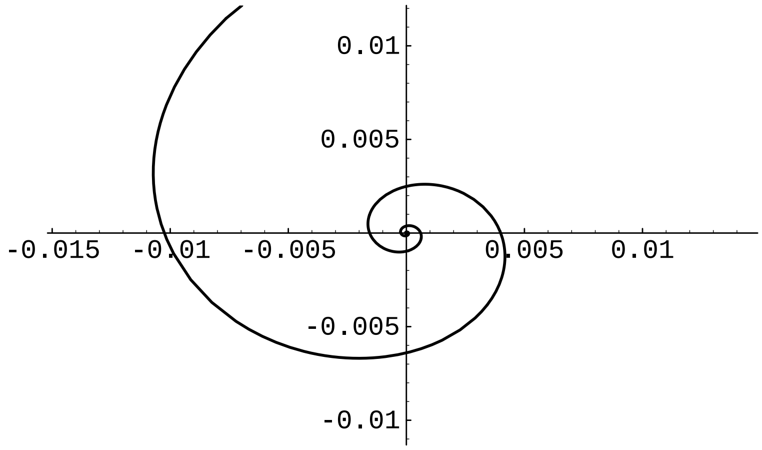
```
vas=NDSolve[{x1'[t]==-x2[t]+x1[t] ( -0.3 - x1[t]^2-x2[t]^2),
x2'[t]==x1[t] + x2[t] ( -0.3 -x1[t]^2 - x2[t]^2),
x1[0]==0.1, x2[0]==0.1},{x1[t],x2[t]},{t,0,30}]
```

```
{{x1[t] -> InterpolatingFunction[{0., 30.}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 30.}, <>][t]}}
```

```
vas={x1[t],x2[t]} /. vas[[1]]
```

```
{InterpolatingFunction[{0., 30.}, <>][t],
  InterpolatingFunction[{0., 30.}, <>][t]}
```

```
ParametricPlot[vas,{t,0,30}]
```




```
-Graphics-
```

Tassa siis origo kayttaytyy kuten stabiili fokus.

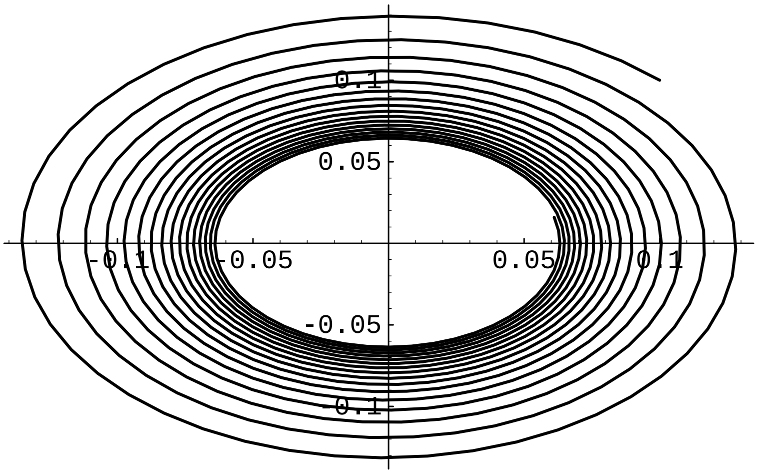
```
vas=NDSolve[{x1'[t]==-x2[t]+x1[t] ( -x1[t]^2-x2[t]^2),
x2'[t]==x1[t] + x2[t] ( -x1[t]^2 - x2[t]^2),
x1[0]==0.1, x2[0]==0.1},{x1[t],x2[t]},{t,0,100}]
```

```
{{x1[t] -> InterpolatingFunction[{0., 100.}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 100.}, <>][t]}}
```

```
vas={x1[t],x2[t]} /. vas[[1]]
```

```
{InterpolatingFunction[{0., 100.}, <>][t],
  InterpolatingFunction[{0., 100.}, <>][t]}
```

```
ParametricPlot[vas,{t,0,100}]
```




```
-Graphics-
```

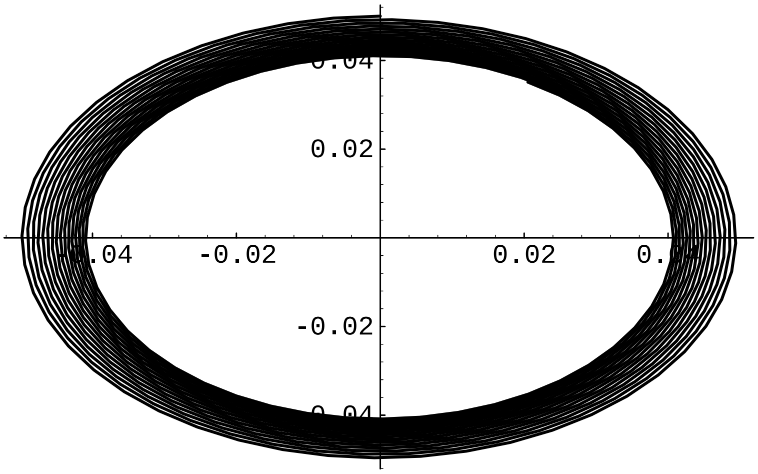
```
vas=NDSolve[{x1'[t]==-x2[t]+x1[t] ( -x1[t]^2-x2[t]^2),
x2'[t]==x1[t] + x2[t] ( -x1[t]^2 - x2[t]^2),
x1[0]==0, x2[0]==0.05},{x1[t],x2[t]},{t,0,100}]
```

```
{{x1[t] -> InterpolatingFunction[{0., 100.}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 100.}, <>][t]}}
```

```
vas={x1[t],x2[t]} /. vas[[1]]
```

```
{InterpolatingFunction[{0., 100.}, <>][t],
  InterpolatingFunction[{0., 100.}, <>][t]}
```

```
ParametricPlot[vas,{t,0,100}]
```




```
-Graphics-
```

Nyt linearisointi ei antanut tulosta, mutta nayttaisi silta, etta ratkaisut paatyva lopulta origoon.

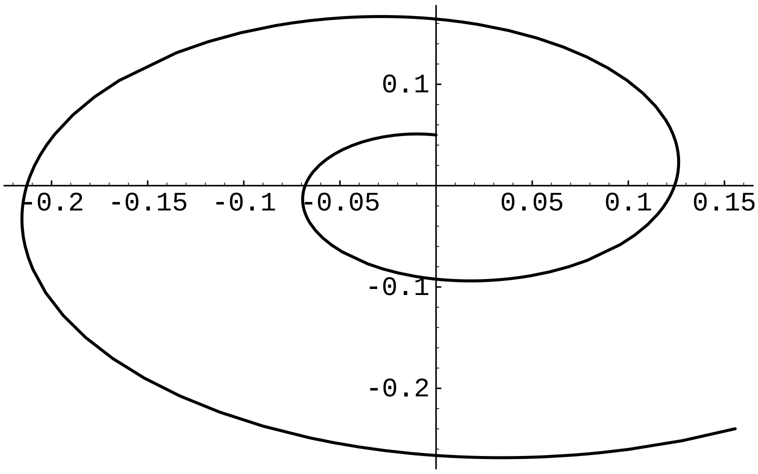
```
vas=NDSolve[{x1'[t]==-x2[t]+x1[t] ( 0.2-x1[t]^2-x2[t]^2),
x2'[t]==x1[t] + x2[t] ( 0.2-x1[t]^2 - x2[t]^2),
x1[0]==0, x2[0]==0.05},{x1[t],x2[t]},{t,0,10}]
```

```
{{x1[t] -> InterpolatingFunction[{0., 10.}, <>][t],
   x2[t] -> InterpolatingFunction[{0., 10.}, <>][t]}}
```

```
vas={x1[t],x2[t]} /. vas[[1]]
```

```
{InterpolatingFunction[{0., 10.}, <>][t],
  InterpolatingFunction[{0., 10.}, <>][t]}
```

```
ParametricPlot[vas,{t,0,10}]
```




```
-Graphics-
```

Tuloksena siis epastabiili fokus.