1.

a) $$0 \le \frac{|\sin n|}{n^2+2n+1} \le \frac{1}{n^2+2n+1} \le \frac{1}{n^2}$$

Koska $\sum_{n=1}^{\infty} \frac{1}{n^2}$ suppenee (p-sarja, $p=2$),

niin myös $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2+2n+1}$ suppenee.

Suppenee tosi myös itseisesti, koska termit ovat positiiviset.

b) $$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$$

Koska $\ln n > 0$, kun $n \ge 2$, on sarja vuorotteleva tekijän $(-1)^n$ takia. (1)

$\ln$ on kasvava, joten $\ln n \le \ln(n+1)$

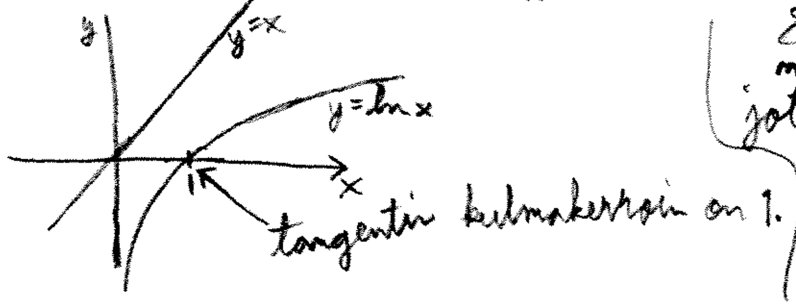
$$\Rightarrow \frac{1}{\ln n} \ge \frac{1}{\ln(n+1)}. \quad (2)$$

$\frac{1}{\ln n} \xrightarrow[n\to\infty]{} 0$, koska $\ln n \xrightarrow[n\to\infty]{} \infty$. (3)

Vuorottelevien sarjojen testin perusteella sarja <u>suppenee.</u>

$$\sum_{n=2}^{\infty} \left| \frac{(-1)^n}{\ln n} \right| = \sum_{n=2}^{\infty} \frac{1}{\ln n}$$

Koska $\ln n \le n$, niin $\frac{1}{\ln n} \ge \frac{1}{n}$. Harmoninen sarja $\sum_{n=1}^{\infty} \frac{1}{n}$ ei suppene, joten $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ <u>ei suppene itseisesti.</u>



2.

$$\bar{r}(t) = \cos(\pi t)\sin(\pi t)\,\bar{i} + \sin^2(\pi t)\,\bar{j} \qquad , 0 \le t \le 1.$$

a)

$$\frac{d\bar{r}}{dt} = \left[-\pi\sin^2(\pi t) + \pi\cos^2(\pi t)\right]\bar{i} +$$
$$+ \left[2\pi\sin(\pi t)\cos(\pi t)\right]\bar{j}$$

$$\left|\frac{d\bar{r}}{dt}\right| = \pi\sqrt{\left[\cos^2(\pi t) - \sin^2(\pi t)\right]^2 + \left[2\sin(\pi t)\cos(\pi t)\right]^2} =$$

$$= \pi\sqrt{[\underbrace{\cos^2(\pi t) + \sin^2(\pi t)}_{=1}]^2} = \pi$$

$$s = \int_0^t \left|\frac{d\bar{r}}{d\tau}\right| d\tau = \pi t$$, koko pituus on π. $(t=1)$

b)

$$\bar{r}(s) = \cos(s)\sin(s)\,\bar{i} + \sin^2(s)\,\bar{j} \qquad , \quad (0 \le s \le \pi)$$

c)

$$\hat{T}(s) = (\cos^2 s - \sin^2 s)\,\bar{i} + 2\sin(s)\cos(s)\,\bar{j}$$

$$\frac{d\hat{T}}{ds} = \left(-2\cos(s)\sin(s) - 2\sin(s)\cos(s)\right)\bar{i} +$$
$$+ \left(2\cos^2 s - 2\sin^2 s\right)\bar{j}$$

$$\kappa(s) = \left|\frac{d\hat{T}}{ds}\right| = 2\sqrt{(-2\sin(s)\cos(s))^2 + (\cos^2 s - \sin^2 s)^2} =$$

$$= 2\sqrt{(\cos^2 s + \sin^2 s)^2} = 2$$

Kaarevuus on siis vakio 2 joten myös t:n funktiona sama vakio. $\underline{\kappa(t) = 2.}$

3.

$$f(x,y) = \frac{xy^3 \cos x}{x^2 + y^6}, \quad (x,y) \neq (0,0)$$

Lähestytään origoa x-akselia pitkin:

$f(x,0) = 0$, joten $\lim_{x \to 0} f(x,0) = 0$

Lähestytään origoa käyrää $x = y^3$ pitkin:

$$f(y^3, y) = \frac{y^3 y^3 \cos y^3}{(y^3)^2 + y^6} = \frac{1}{2} \cos y^3 \xrightarrow[y \to 0]{} \frac{1}{2}$$

f:llä ei siis ole raja-arvoa origossa

4.

$$f(x,y) = x^2 y + \frac{y}{x^2+1}$$

a)

$$\nabla f(x,y) = \left[2xy - \frac{2xy}{(x^2+1)^2}\right]\bar{i} + \left[x^2 + \frac{1}{x^2+1}\right]\bar{j}$$

$$\nabla f(1,1) = \underline{\underline{\frac{3}{2}\bar{i} + \frac{3}{2}\bar{j}}}$$ on normaalivektori.

b)

$$g(x,y,z) := z - f(x,y)$$

$$z = f(x,y) \Leftrightarrow g(x,y,z) = 0$$

Tangenttitason normaalivektori:

$$\nabla g(x,y,z) = -\frac{\partial f}{\partial x}\bar{i} - \frac{\partial f}{\partial y}\bar{j} + \bar{k}$$

$$\nabla g(1,1,\tfrac{3}{2}) = -\frac{3}{2}\bar{i} - \frac{3}{2}\bar{j} + \bar{k}$$

Yhtälö:

$$-\frac{3}{2}(x-1) - \frac{3}{2}(y-1) + 1\cdot\left(z - \frac{3}{2}\right) = 0$$

$$\Leftrightarrow \underline{\underline{-\frac{3}{2}x - \frac{3}{2}y + z = -\frac{3}{2}}}$$