HUT , Institute of mathematics Gripenberg
Mat-1.196 Mathematics of neural networks
Exercise 9
12.3–21.3.2002

1. Assume that $\mathbf{x} \in \mathbb{R}^d$. Calculate the gradient of the function

$$f(\mathbf{w}) = \tfrac{1}{2}|\mathbf{x} - (\mathbf{w} \cdot \mathbf{x})\mathbf{w}|^2, \quad \mathbf{w} \in \mathbb{R}^d,$$

at a point where $|\mathbf{w}| = 1$.

2. Assume that $\mathbf{w} \in \mathbb{R}^d$ is such that $|\mathbf{w}| = 1$. If $\mathbf{x} \in \mathbb{R}^d$, define

$$\mathbf{w}^* = \frac{1}{|\mathbf{w} + \eta(\mathbf{w} \cdot \mathbf{x})\mathbf{x}|}(\mathbf{w} + \eta(\mathbf{w} \cdot \mathbf{x})\mathbf{x}),$$

so that $|\mathbf{w}^*| = 1$. Show that

$$\mathbf{w}^* = \mathbf{w} + \eta(\mathbf{w} \cdot \mathbf{x})(\mathbf{x} - (\mathbf{w} \cdot \mathbf{x})\mathbf{w}) + O(\eta^2).$$

3. Assume that $x \in \mathbb{R}^{d\times 1}$. Find the gradient of the function

$$f(W) = \tfrac{1}{2}|X - W^{\mathrm{T}}WX|^2, \quad W \in \mathbb{R}^{m\times d},$$

at a point where $WW^{\mathrm{T}} = I$.