HUT , Institute of mathematics Gripenberg
Mat-1.196 Mathematics of neural networks
Exercise 7
26.2–6.3.2002

1. Let $\mathbf{u} \in \mathbb{R}^d$ be such that $|\mathbf{u}| = 1$. Define the Radon-transform $P_{\mathbf{u}} f$ as follows:

$$(P_{\mathbf{u}} f)(t) = \int_{\mathbb{R}^{d-1}} f(t\mathbf{u} + U^{\perp}\mathbf{s})\,\mathrm{d}\mathbf{s},$$

where $U^{\perp}$ is a $d \times (d-1)$ matrix with columns that form an orthonormal basis for the subspace of vectors in $\mathbb{R}^d$ orthogonal to $\mathbf{u}$. Show that if $f \in L^1(\mathbb{R}^d)$, then $P_{\mathbf{u}} f \in L^1(\mathbb{R})$ and

$$\widehat{P_{\mathbf{u}} f}(\underline{\omega}) = \hat{f}(\underline{\omega}\mathbf{u}).$$

2. Let H be a separable Hilbert space, let $(f_n)_{n=1}^{\infty}$ be a frame in H, and let $Tf = \sum_{n=1}^{\infty} \langle f, f_n \rangle f_n$. Assuming that it has been shown that T is a bounded selfadjoint operator with bounded inverse, show that $(T^{-1} f_n)_{n=1}^{\infty}$ is a frame as well.

3. Let H be a separable Hilbert space and let $(f_n)_{n=1}^{\infty}$ be a frame in H. Does it follow that

$$\sup_{n \geq 1} \|f_n\| < \infty?$$