MS-A0401 Foundations of discrete mathematics

Gripenberg, Potka, Pöllänen, Tuhkanen, Ääri

Exercise 6

12.10–16.10.2015, week 42

Return your solutions to the P-questions and answer the S-questions not later than 19.10.2015 at 16.

**Remember to write your name, student number and group!**

**P1.** A group of students plan to take part in exams in the courses $C_1, \dots, C_6$ as follows:

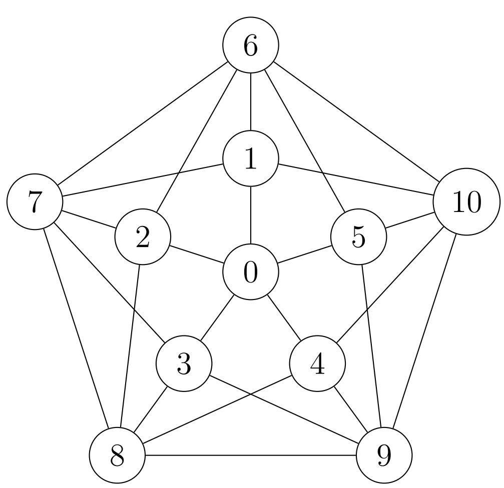
| | | | |
|---|---|---|---|
| Student A | $C_1$ | $C_3$ | |
| Student B | $C_1$ | $C_4$ | $C_5$ |
| Student C | $C_2$ | $C_6$ | |
| Student D | $C_2$ | $C_3$ | $C_6$ |
| Student E | $C_3$ | $C_4$ | |
| Student F | $C_3$ | $C_5$ | |
| Student G | $C_5$ | $C_6$ | |

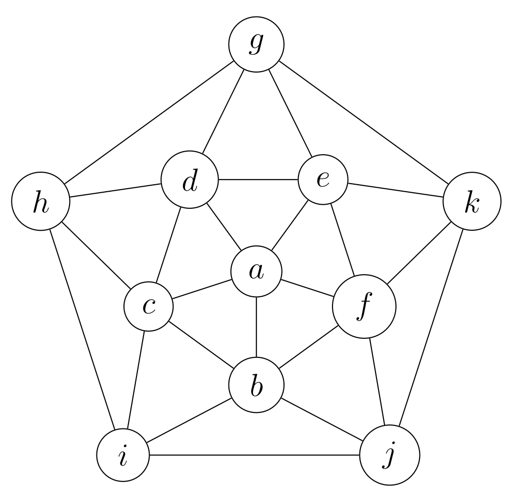
Draw a graph with nodes $C_j$, $j = 1, 2, \dots 6$ so that there is an edge between $C_j$ and $C_k$ if and only if there is at least one student that plans to take part in the exams in courses $C_j$ and $C_k$. Determine the chromatic number of this graph, that is the smallest number of colours needed to coulour the nodes so that if there is an edge between two nodes, then they get different colours. What does this number tell us in this case?

**P2.**

(a) Determine the number of neighbours for each node in the graphs below and write, for each graph, these numbers as a sequence in nonincreasing order. Are these sequences identical?

(b) Are the graphs below isomorphic? Give reasons for your answer!




**P3.** The nodes of the graph $[V, E]$ are the lists $[s_1, s_2, s_3]$ where $s_j \in \{1, 2\}$, $j = 1, 2, 3$. There is an undirected edge between the nodes $[s_1, s_2, s_3]$ and $[t_1, t_2, t_3]$ if and only if $|s_1 - t_1| + |s_2 - t_2| + |s_3 - t_3| = 1$. Draw this graph and put the nodes in an order so that the greedy colouring algorithm uses 3 colours.

*Hint: Draw for example first the nodes* $[1, s_2, s_3]$ *and then the nodes* $[2, s_2, s_3]$.

**P4.** Let $[V, E]$ be an undirected graph which is simple (that is $\{v, v\} = \{v\} \notin E$ for all $v \in V$) and where $V = \{1, 2, 3, 4, 5\}$. Let $n(v)$ be the number of neighbours of node $v$ (i.e., the degree of the node). Which of the following lists can be the list $[n(1), n(2), n(3), n(4), n(5)]$ and which cannot?

(a) $[1, 3, 3, 4, 2]$, (b) $[2, 2, 2, 4, 4]$, (c) $[1, 2, 3, 4, 4]$

Draw the graph if it is possible and explain why it is not possible otherwise.

*Hint: Since the graph is undirected and simple you can express the sum* $\sum_{j=1}^{5} n(j)$ *with the aid of the number* $|E|$ *of edges.*

**P5.** According to the definition a graph is a forest if it is simple (there is never an edge from a node to itself) and from each edge there is at most one simple path to each other node, that is, there are no simple cycles in the graph.

(a) Show that if in a forest there is a node with exactly one neighbour then there is another node that has exactly one neighbour as well.

(b) Show that if in house there is only one external door, then there is a room in the house with an odd number of doors.

*Hint: In part (b) you can construct a suitable graph and then remove the edges on simple cycles until you get a forest and then apply the result from part (a).*