Aalto University

MS-A0401 Foundations of discrete mathematics

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Exercise 3

21–25.9.2015, week 39

Return your solutions to the P-questions and answer the S-questions not later than 28.9.2015 at 16.

Remember to write your name, student number and group!

P1. The number of students that have signed up for an exercise group in a course is n. Show that at least two of the students have earlier met the same number of those who signed up for this course on some other course. (Here it is assumed that the ”meet” relation is symmetric but irrefelxive, i.e., no one has met themselves on some course.)

Hint: If some student has met everyone else, how many have the others at least met?

P2. A person decides to choose 100 of the days in 2016 when she will run a 10 km route, 200 days when she will run a 5 km route and 66 when she will not do any extra exercises. In how many ways can she make these choices?

(a) Give the answer as a multinomial coefficient.

(b) Calculate the answer using the product rule so that she at the first stage chooses the 100 days when she will run the 10 km route and then among the remaining days chooses the 200 ones when she will run the 5 km route and give your answer using binomial coefficients.

(c) Show that the answers in parts (a) and (b) are the same without actually calculating them.

P3. The set X has n elements. A relation W in X is asymmetric if $[x, y] \in W \to [y, x] \notin W$ for all x and $y \in X$. Determine how many asymmetric relations W there are in X for example by first answering the following questions where it is assumed that $X = \{x_1, x_2, \dots, x_n\}$:

(a) Does $[x_i, x_i] \in W$ or $[x_i, x_i] \notin W$ hold if W is asymmetric?

(b) How many alternatives are there when one considers whether the pairs $[x_i, x_j]$ and/or $[x_j, x_i]$ belong to the relation W if $1 \leq i < j \leq n$ anda W is asymmetric?

(c) How many pairs of integers $[i, j]$ are there so that $1 \leq i <\leq n$?

(d) Calculate the number of asymmetric relations using the results from parts (a), (b), and (c) together with the product principle.

P4.

(a) In how many ways can one order the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 so that all of the odd numbers come immediately after each other?

(b) In how many ways can one order the numbers 1, 2, 3, 4, 5, 6, 7, 8 if it is required that no odd number comes immediately after another odd number?

Answer: (a) 14400, (b) 2880

P5. Show that $|\mathbb{N}_0| < |\{0,1\}^{\mathbb{N}_0}|$ where $\{0,1\}^{\mathbb{N}_0}$ is the set of all functis $f : \mathbb{N}_0 \to \{0,1\}$ in the following way:

(a) Construct an injection: $\mathbb{N}_0 \to \{0,1\}^{\mathbb{N}_0}$.

(b) Show that there is no surjection $h : \mathbb{N}_0 \to \{0,1\}^{\mathbb{N}_0}$.

Hint: In part (a) there are many simple (?) alternatives and in part (b) you can assume that such a surjection exists and derive a contradiction by construcing a function f that belongs to the set $\{0,1\}^{\mathbb{N}_0}$ but $f \neq h(n)$ for all $n \in \mathbb{N}_0$. If $n \in \mathbb{N}_0$ then $h(n)$ is a function $\mathbb{N}_0 \to \{0,1\}$, i.e., $h(n)(j) \in \{0,1\}$ for all $j \geq 0$. Define the function f with the aid of the numbers $h(n)(n)$. When h, as here, is a function that takes it values in a set of functions you can instead of $h(n)$ write h_n if that makes the situation easier.