

1. Exercise E9.1 on page 231.

2. Exercise E9.2 on page 231.

3. Prove Theorem 9.7.(g) (cDOM) on page 88, assuming (cFATOU) known.

4. Let the random variable  $X \in L^2(\Omega, \mathcal{F}, P)$  and let  $\mathcal{G} \subset \mathcal{F}$  ( $\mathcal{G}$  a sub- $\sigma$ -algebra of  $\mathcal{F}$ ). Define

$$\text{Var}(X | \mathcal{G}) = \mathbb{E}[(X - \mathbb{E}(X | \mathcal{G}))^2 | \mathcal{G}].$$

Show that

$$\text{Var}(X) = \mathbb{E}(\text{Var}(X | \mathcal{G})) + \text{Var}(\mathbb{E}(X | \mathcal{G})).$$

5. Let  $X, Y$  be two independent random variables taking values in  $\{0, 1\}$  such that  $\mathbb{P}(X = 1) = p$  and  $\mathbb{P}(Y = 1) = p$ ,  $p \in (0, 1)$ . Define

$$Z = 1_{\{X+Y=0\}}.$$

Compute  $\mathbb{E}(X|Z)$  and  $\mathbb{E}(Y|Z)$ . Are these independent?