Mat-5.3741 Theory of Elasticity (5 cp) L **Spring 2007**
Stenberg/Juntunen

Exercise 8

Problem 1
A scalar value function $u : \Omega \to \mathbb{R}$ is called subharmonic if

$$\Delta u \geq 0.$$

A well-known result from analysis is that a subharmonic function attains its maximum value on the boundary $\partial\Omega$. Consider the shear force vector

$$\boldsymbol{\tau} = 2G\alpha \left(\frac{\partial \phi}{\partial x_2} - \frac{\partial \phi}{\partial x_1} \right)$$

in which ϕ is the stress function. Prove, using the theorem above, that $|\boldsymbol{\tau}|$ attains its maximum value at the boundary $\partial\Omega$.

Problem 2
Compute approximately the torsional rigidity for the square $[0, a] \times [0, a]$ and the triangle

$$\{(x, y) \,|\, x \geq 0, y \geq 0, x + y \leq a\}$$

using the Galerkin method with only one basis function, i.e. the lowest polynomial in x and y that vanish on the boundary.
(Compare to the exact values. For the square see the previous exercise. For the triangle the exact value will be computed by Antti H.)

Problem 3 **(home exercise)**
Consider a thin tube with central radius R and thickness t and the same tube cut open. Derive the approximate torsional rigidities for both cases. Let the tube be loaded with the moment M. What are the maximal shear forces in the tube for the two cases?