Mat-1.3379 Dynaamisten systeemien erikoiskurssi **Eirola/Hannukainen**
Harjoitus 2, 2.2.2006 12-14 Y338C

Harjoitus 2 on tietokoneharjoitus. Tehtäviä tehdään yhdessä assistentin kanssa tietokoneluokassa ja joistain tehtävistä palautetaan lyhyt selostus seuraavan viikon harjoituksiin mennesä.

1. (Problem 1.2. p. 55) Consider this nonlinear system: $x^2 + x + y^2 - y - 1 = 0$. Use Gauss-Newton with at least two different choices for the right inverse to determine where and how you converge. Repeat replacing -1 with $1/2$.

2. (Problem 4.1. p. 64) Implement the `predcorr` function; you need to write the functions `lxb` and `uxb` to solve lower triangular and upper triangular systems. Then, solve the problems below, making sure that you do not step past $\alpha = 1$
   (a) Using the embedding of Example 4.2, find a root of $\boldsymbol{f}(\boldsymbol{x}) = 0$, where $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x} - \boldsymbol{g}(\boldsymbol{x})$, and
   $$g_i(\boldsymbol{x}) = (i + \sum_{k=1}^{n} x_k^3)/(2n)\,,\ \ i = 1, 2, \ldots, n\,,$$
   for $n = 10$.
   (b) With the same embedding as in Example 4.2, find a root of $\boldsymbol{f}(\boldsymbol{x}) = 0$, where $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x} - \boldsymbol{g}(\boldsymbol{x})$, and now
   $$g_1(\boldsymbol{x}) = 1 + x_1 - \textstyle\prod_{j=1}^{n} x_j\ , \quad g_i(\boldsymbol{x}) = n + 1 - \sum_{j=1}^{n} x_j\ , \quad i = 2, \ldots, n\,.$$
   Do it for $n = 10$ and $n = 25$. (Exact solution here is $\boldsymbol{x} = (1, \ldots, 1)$). Also, estimate the length of the $\boldsymbol{x}(\alpha)$–curve, $\alpha \in [0, 1]$.
   (c) Try solving the problems in (a)-(b) with the embedding of Example 4.1. If successful, compare your results with those of (a)-(b).

   Return (by 9.2.) the number of iterations, plot out of the error $|\boldsymbol{f}(x^k)|$ as a function of iterations and other observed data. The `predcorr.m` - function can be found from the course homepage.

3. (Problem 4.3. p. 69) Building on `predcorr` and Remark 4.4, write a program implementing either the approach of Example 4.5 or of Example 4.6 to solve $\boldsymbol{f}(\boldsymbol{x}, \alpha) = 0$. If you implement the Gauss-Newton approach of Example 4.6, use the pseudo-inverse and proceed in a stationary (Gauss) Newton way. Then, use your program to solve the two problems below. Be aware that it is tricky with this approach to make sure one does not step past $\alpha = 1$: In your implementation, make sure that you end at $\alpha = 1$ (within a reasonable bound).
   (a) We need to solve $\boldsymbol{f}(\boldsymbol{x}, \alpha) = 0$, where
   $$\boldsymbol{f}(\boldsymbol{x}, \alpha) = \begin{bmatrix} -x_1 + \alpha(1 - x_1)e^{x_2} \\ -3x_2 + 14\alpha(1 - x_1)e^{x_2} \end{bmatrix}\,.$$
   You need to trace the solution from $\alpha = 0$ (where $\boldsymbol{x} = 0$) to $\alpha = 1$. Produce three plots of the solution: (i) $(\alpha, \|\boldsymbol{x}\|)$, (ii) (α, x_1), and (iii) (α, x_2) and observe that the solution curve is not parametrizable in α.

(b) This is a homotopy problem in which the embedding of Example 4.1 is used. We have $\boldsymbol{f}(\boldsymbol{x},\alpha) = \boldsymbol{g}(\boldsymbol{x}) - (1-\alpha)\boldsymbol{g}(\boldsymbol{x}^0)$, where

$$\boldsymbol{g}(\boldsymbol{x}) = (\boldsymbol{G}_{\boldsymbol{x}}(\boldsymbol{x}))^T \boldsymbol{G}(\boldsymbol{x})\,, \quad \boldsymbol{G}(\boldsymbol{x}) = \begin{bmatrix} 10(x_2 - x_1^2) \\ 1 - x_1 \\ 3\sqrt{10}(x_4 - x_3^2) \\ 1 - x_3 \\ \sqrt{10}(x_2 + x_4 - 2) \\ (x_2 - x_4)/\sqrt{10} \end{bmatrix}.$$

At $\alpha = 0$, the solution is $\boldsymbol{x}^0 = (-3,-1,-3,-1)^T$, and we need the solution for $\alpha = 1$. (Hint: exact solution is $(1,1,1,1)^T$ and the problem is very hard close to $\alpha = 1$).