Teht 11/mplV014.mw

```
> restart :
  with(plots) :
  setoptions3d(axes = boxed)
```

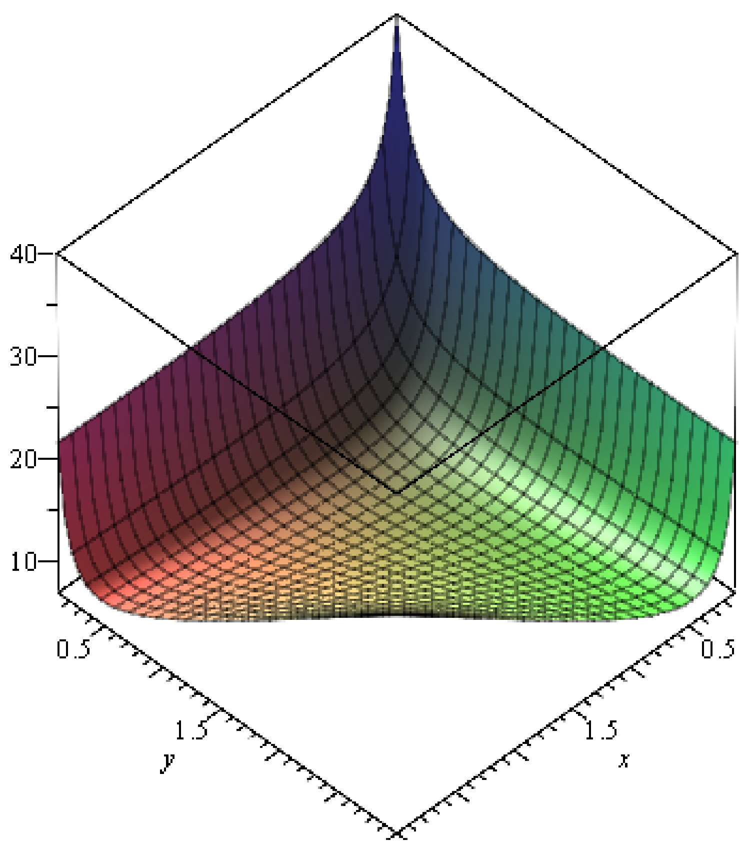
Laatikko

$h := (x, y) \rightarrow 3 \cdot x \cdot y + 2 \cdot V \cdot \left(\frac{1}{x} + \frac{1}{y}\right)$

$$(x, y) \rightarrow 3\, x\, y + 2\, V \left(\frac{1}{x} + \frac{1}{y}\right) \quad (1.1)$$

```
>
V := 1; plot3d(h(x, y), x = .1 ..3, y = .1 ..3)
```

1




```
> V :='V'
```

$$V := V \quad (1.2)$$

$d1 := diff(h(x, y), x)$

$$3\,y - \frac{2\,V}{x^2} \tag{1.3}$$

$d2 := diff(h(x,y), y)$

$$3\,x - \frac{2\,V}{y^2} \tag{1.4}$$

$solve(\{d1 = 0, d2 = 0\}, \{x, y\})$

$$\{x = RootOf(-2\,V + 3\,\_Z^3), y = RootOf(-2\,V + 3\,\_Z^3)\} \tag{1.5}$$

$allvalues(\%)$

$$\left\{x = \frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}, y = \frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}\right\}, \left\{x = \frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}\,(-1)^{2/3}, y = \frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}\,(-1)^{2/3}\right\}, \left\{x = -\frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}\,(-1)^{1/3}, y = -\frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}\,(-1)^{1/3}\right\} \tag{1.6}$$

$xy := \%[1]$

$$\left\{x = \frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}, y = \frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}\right\} \tag{1.7}$$

$x0 := subs(xy, x)$

$$\frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3} \tag{1.8}$$

$y0 := subs(xy, y)$

$$\frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3} \tag{1.9}$$

$minimi := [x0, y0]; evalf(minimi)$

$$\left[\frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}, \frac{1}{3}\,2^{1/3}\,3^{2/3}\,V^{1/3}\right]$$

$$[0.8735804646\,V^{1/3}, 0.8735804646\,V^{1/3}] \tag{1.10}$$

Kun $x \to \infty$ tai $y \to \infty$, niin $h \to \infty$, samoin, kun lähestytään x- tai y-akselia. Siten kaikki minimit ovat gradientin nollakohtia.

Huom! Voidaan toki laskea oikein mukavasti myös *Lagrangen kertojatyylillä*.