H2T13R, Rungen ilmiö

> *restart* :

> *with*(*plots*) :

> *linspace* := (*a*, *b*, *n*) → [*seq*(*a* + *iii* * (*b* − *a*) / (*n* − 1), *iii* = 0 ..*n* - 1)]

$$linspace := (a, b, n) \rightarrow \left[seq\left(a + \frac{iii\ (b-a)}{n-1}, iii = 0\ ..n-1\right)\right] \tag{1}$$

> $g := x \rightarrow \frac{1}{1+x^2}$

$$g := x \rightarrow \frac{1}{1+x^2} \tag{2}$$

> *xd* := *linspace*(-5, 5, 10)

$$xd := \left[-5, -\frac{35}{9}, -\frac{25}{9}, -\frac{5}{3}, -\frac{5}{9}, \frac{5}{9}, \frac{5}{3}, \frac{25}{9}, \frac{35}{9}, 5\right] \tag{3}$$

> *yd* := *map*(*g*, *xd*)

$$yd := \left[\frac{1}{26}, \frac{81}{1306}, \frac{81}{706}, \frac{9}{34}, \frac{81}{106}, \frac{81}{106}, \frac{9}{34}, \frac{81}{706}, \frac{81}{1306}, \frac{1}{26}\right] \tag{4}$$

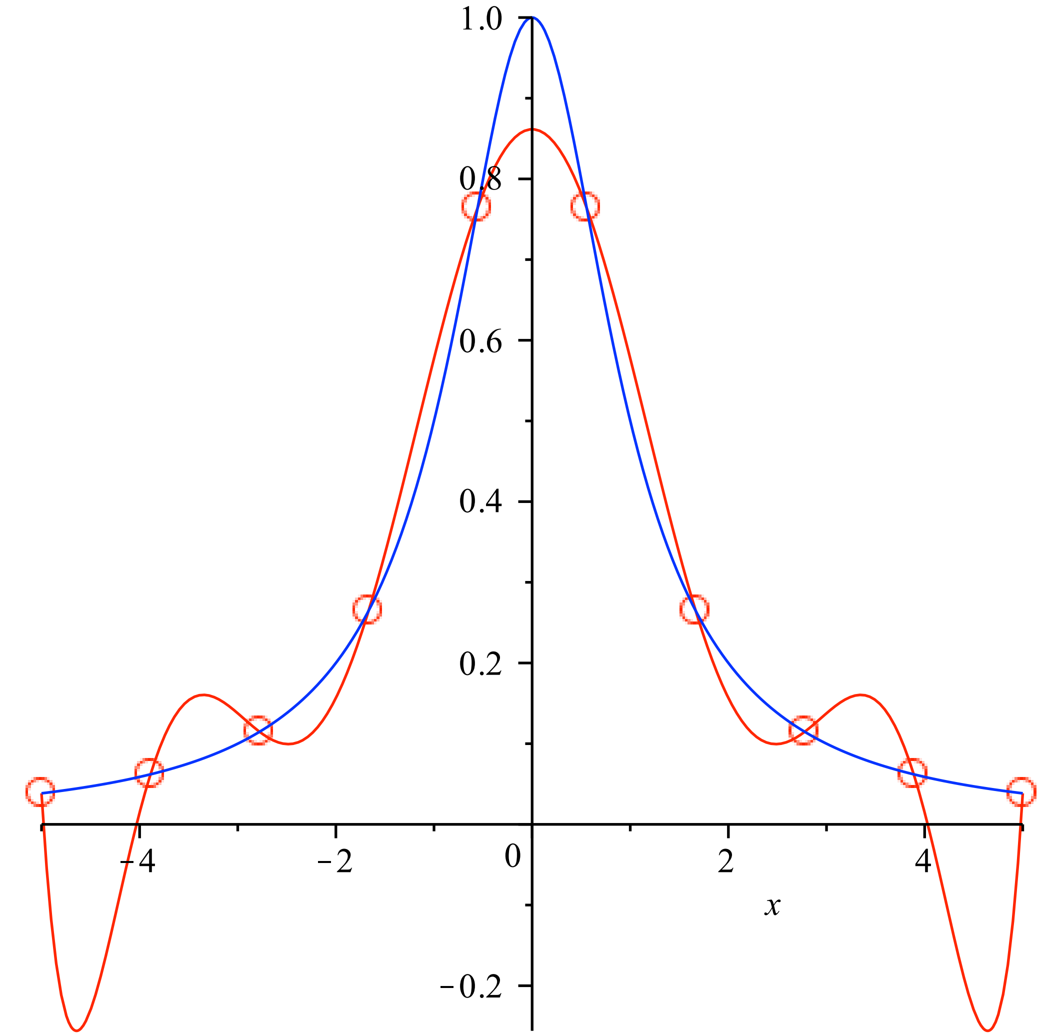
> *with*(*CurveFitting*)

[*ArrayInterpolation*, *BSpline*, *BSplineCurve*, *Interactive*, *LeastSquares*, *PolynomialInterpolation*, *RationalInterpolation*, *Spline*, *ThieleInterpolation*] **(5)**

> *p* := *PolynomialInterpolation*(*xd*, *yd*, *x*)

$$p := \frac{4782969}{86398461344} x^8 - \frac{124180047}{43199230672} x^6 + \frac{530979543}{10799807668} x^4 - \frac{14274621297}{43199230672} x^2 + \frac{74435570719}{86398461344} \tag{6}$$

> *display*(*plot*(*p*, *x* = -5 ..5), *plot*(*xd*, *yd*, *style* = *point*, *symbol* = *circle*, *symbolsize* = 20), *plot*(*g*(*x*), *x* = -5 ..5, *color* = *blue*))

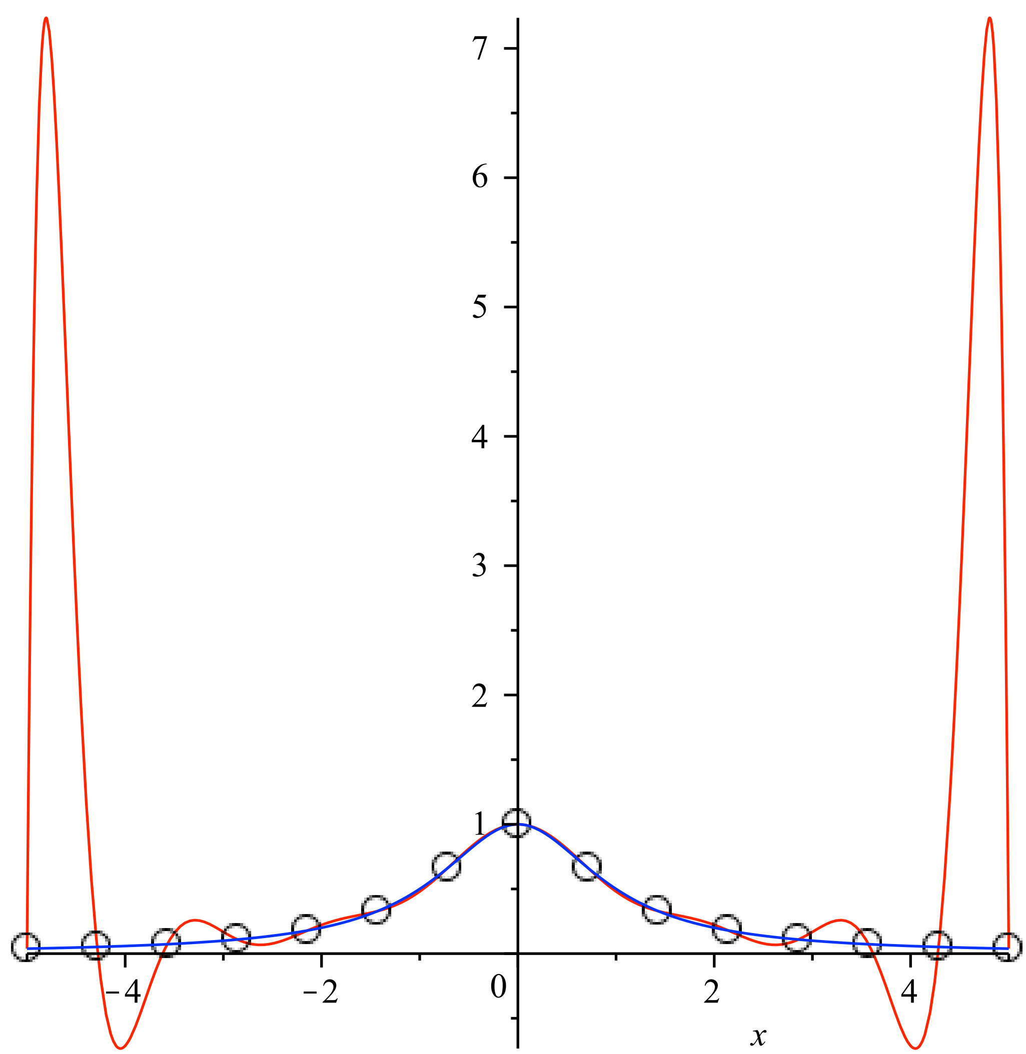


> $gkuva := plot(g(x), x=-5..5, color=blue)$

$$gkuva := PLOT(...) \quad (7)$$

> $n := 15 :$

> $xd := linspace(-5, 5, n) : yd := map(g, xd) : p := PolynomialInterpolation(xd, yd, x) :$
> $display(plot(p, x=-5..5), plot(xd, yd, style=point, color=black, symbol=circle, symbolsize=20), gkuva)$



Tässä jo näkyy (ja vakuuttavammin jatkamalla kokeiluja eri n:n arvoilla), että p seuraa g-funktiota yhä tarkemmin välin keskiosalla, mutta räjähtää reunojen läheisyydessä.

Ei ole niin ollen kaukaa haettu ajatus valita interpolointiopisteet niin, että ne kasautuvat kohti reunoja. Tsebyshev-pisteet normeerattuna välille [-5,5] muodostavat optimaalisen pisteistön.

> $N := 15 : xT := \left[seq\left(5\cdot\cos\left(\frac{(N-j)\cdot\pi}{N}\right), j = 0 ..N\right)\right]$

$$xT := \left[-5, -5\cos\left(\frac{1}{15}\pi\right), -5\cos\left(\frac{2}{15}\pi\right), -5\cos\left(\frac{1}{5}\pi\right), -5\cos\left(\frac{4}{15}\pi\right), -\frac{5}{2}, -5\cos\left(\frac{2}{5}\pi\right), -5\cos\left(\frac{7}{15}\pi\right), 5\cos\left(\frac{7}{15}\pi\right), 5\cos\left(\frac{2}{5}\pi\right), \frac{5}{2}, 5\cos\left(\frac{4}{15}\pi\right), 5\cos\left(\frac{1}{5}\pi\right), 5\cos\left(\frac{2}{15}\pi\right), 5\cos\left(\frac{1}{15}\pi\right), 5\right] \quad \textbf{(8)}$$

> $plot(xT, [0\$16], style = point, axes = none);$

> $n := 15:$

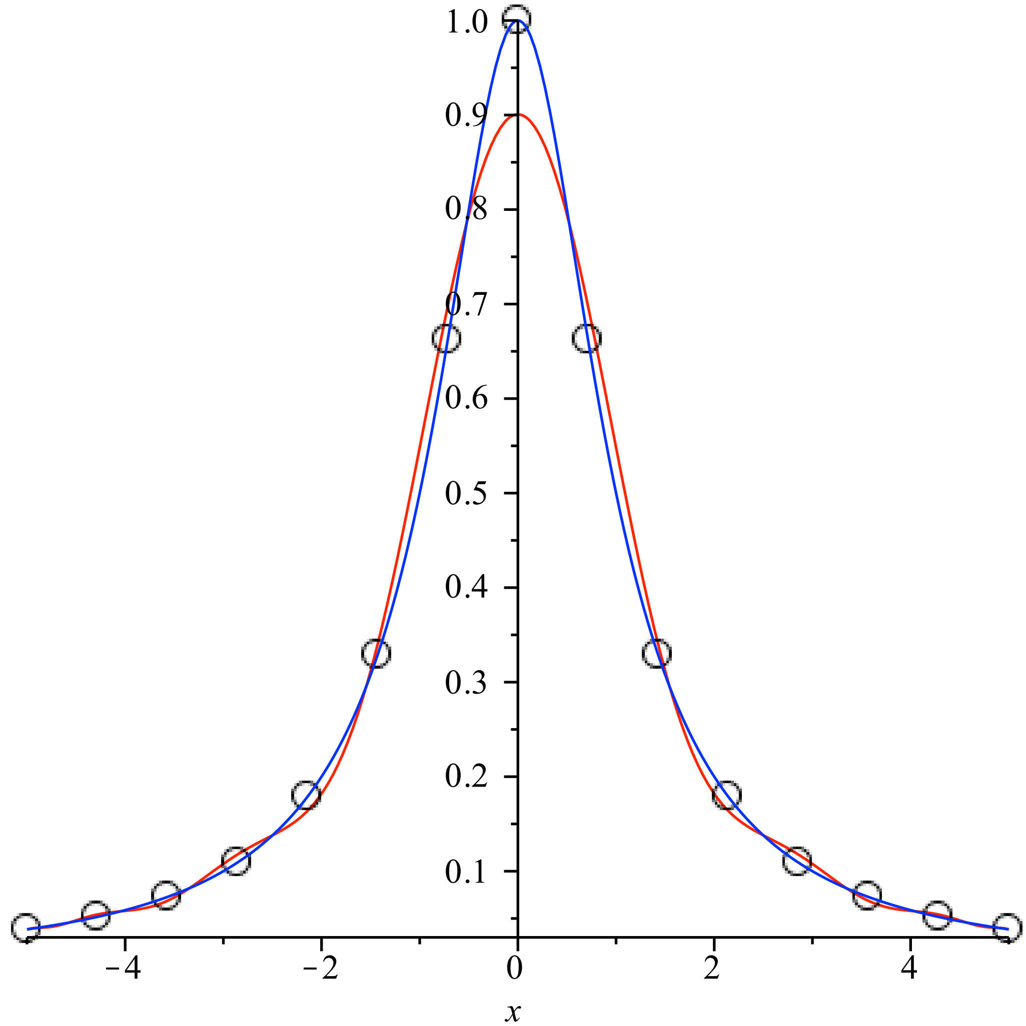
> $xT := \left[seq\left(5 \cdot \cos\left(\frac{(N-j)\cdot \pi}{N}\right), j = 0..N\right)\right] : xT := evalf(xT):$

> $yT := map(g, xT):$

> $p := PolynomialInterpolation(xT, yT, x)$

$$p := -5.3\ 10^{-9}\ x - 0.4517432189\ x^2 + 3.309\ 10^{-8}\ x^3 + 0.1135721973\ x^4 - 2.131\ 10^{-8}\ x^5 - 0.01532466773\ x^6 + 5.340\ 10^{-9}\ x^7 + 0.001167269775\ x^8 - 6.115\ 10^{-10}\ x^9 - 0.00005030077847\ x^{10} + 3.6681\ 10^{-11}\ x^{11} + 0.000001143774066\ x^{12} - 1.07898\ 10^{-12}\ x^{13} - 1.066454944\ 10^{-8}\ x^{14} + 1.241424508\ 10^{-14}\ x^{15} + 0.9006781150 \quad \textbf{(9)}$$

> $display(plot(p, x = -5..5), plot(xd, yd, style = point, color = black, symbol = circle, symbolsize = 20), gkuva)$



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Tuo temppu tekikin ihmeitä!