Introduction to MATLAB

**HOMEWORK 1/Accumulating savings with the fixed balance strategy**

You make an investment of $I = 500EUR$ every 6 months. The investments are balanced between bonds and stocks so that the goal is to keep the bond-stock balance close to $\beta$ — $(1-\beta)$ (proportion $\beta$ stocks, $1-\beta$ bonds). How do your savings accumulate in 20 years with this strategy? Give some statistics (mean, deviance).

Make the following assumptions about the markets and your saving procedure:

- Let time $t$ be discretized to 6 month time steps,

  $$0 = t_0, t_1, t_2, \ldots.$$

  Discretize your savings in stocks as, and in bonds,

  $$s = s_0, s_1, \ldots, \quad b = b_0, b_1, \ldots,$$

  correspondingly.

- Six month market movement. The value of your previous (upto time $t_{i-1}$) stock investment at time $t = t_i$ is

  $$p_s s_{i-1},$$

  where $p_s \sim N(\mu_s, \sigma_s^2)$ is random,

  $$\mu_s = 1.04, \quad \sigma_s = 0.15.$$

  Similarly, the value at time $t_i$ for the previous bond investment is

  $$p_b b_{i-1}, \quad p_b \sim N(\mu_b, \sigma_b^2), \quad \mu_s = 1.02, \quad \sigma_b = 0.02.$$

- Investment decision. At time $t_i$ the entire investment $I$ is done either to stocks or to bonds, depending in which way the balance will be closer to the desired bond-stock balance $\beta$, i.e., either

  $$\begin{cases} s_i = p_s s_{i-1} + I \\ b_i = p_b b_{i-1}, \end{cases} \quad \text{or,} \quad \begin{cases} s_i = p_s s_{i-1} \\ b_i = p_b b_{i-1} + I. \end{cases}$$

**Programming**

- Write a function `investmentrealization.m` with which you can calculate one realization of the evolution of the investment

$$t \mapsto s(t), \quad t \mapsto b(t), \quad t \mapsto s(t) + b(t), \quad t \in [0, 20].$$

- Write m-file that calls `investmentrealization`, say 10000 times, and computes the statistics (mean, deviance) for the final bond and stock investment values. Variate $\beta$.

- Visualize you calculations.

Extra question (not obligatory):
How do the mean and the deviance change when $\beta$ increases? What is the optimal $\beta$ if you ask that your probability to lose money at the end (final value is less than $40 \times 500$EUR) has to be at most 5%? Here, optimal means maximal mean.