
Abstract: We show that every operator that acts between two nonseparable Hilbert spaces can be “block diagonalized”, where each diagonal block acts between two separable Hilbert spaces. Analogous results hold for operator-valued $\mathcal{H}^\infty$, $L^\infty_{\text{strong}}$, $\mathcal{H}^p_{\text{strong}}$ and $L^p_{\text{strong}}$ functions and others. Using these results, several theorems about representation, interpolation, invertibility, factorization etc., which have previously been known only for separable Hilbert spaces, can now be generalized to arbitrary Hilbert spaces. We generalize several results often needed in systems and control theory, including the Nehari (or Page) Theorem, the Adamjan–Arov–Krein Theorem, the Hartman Theorem, the Lax–Halmos Theorem, Tolokonnikov’s Lemma and the inner-outer factorization. We present our results both for the unit circle/disc and for the real line/half-plane.


Keywords: Orthogonal subspaces, strong Hardy spaces of operator-valued functions, strongly essentially bounded functions, shift-invariant subspaces, translation-invariant operators, inner functions, left invertibility.

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ISSN 0784-3143
TKK, Espoo 2007

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