Helsinki University of Technology, Institute of Mathematics, Research Reports

Teknillisen korkeakoulun matematiikan laitoksen tutkimusraporttisarja

Espoo 2004 A475

# APPLYING MATHEMATICAL FINANCE TOOLS TO THE COMPETITIVE NORDIC ELECTRICITY MARKET

**Iivo Vehviläinen**

Helsinki University of Technology, Institute of Mathematics, Research Reports
Teknillisen korkeakoulun matematiikan laitoksen tutkimusraporttisarja
Espoo 2004 A475

# APPLYING MATHEMATICAL FINANCE TOOLS TO THE COMPETITIVE NORDIC ELECTRICITY MARKET

**Iivo Vehviläinen**

**Dissertation for the degree of Doctor of Science in Technology to be presented with due permission of the Department of Engineering Physics and Mathematics, for public examination and debate in Auditorium K at Helsinki University of Technology (Espoo, Finland) on the 3rd of December, 2004, at 12 o'clock noon.**

Helsinki University of Technology
Department of Engineering Physics and Mathematics
Institute of Mathematics

**Iivo Vehviläinen**: *Applying mathematical finance tools to the competitive Nordic electricity market*; Helsinki University of Technology, Institute of Mathematics, Research Reports A475 (2004).

**Abstract:** *Deregulation of electricity industry has introduced competitive electricity markets that are similar to financial markets. Application of mathematical finance requires careful consideration of special characteristics of electricity markets. This thesis models competitive electricity markets with the methods of mathematical finance. Fundamental problems of finance are market price modelling, derivative pricing, and optimal portfolio selection. The same questions arise in competitive electricity markets. Transparently applicable theoretical models will be more important in the increasingly competitive economy.*

**AMS subject classifications:** 91B28, 60G35

**Keywords:** mathematical finance, electricity markets, risk management, derivative pricing, stochastic modelling

**Correspondence**

Iivo.Vehvilainen@iki.fi

ISBN 951-22-7323-3
ISSN 0784-3143

Helsinki University of Technology
Department of Engineering Physics and Mathematics
Institute of Mathematics
P.O. Box 1100, 02015 HUT, Finland
email:math@hut.fi http://www.math.hut.fi/

# Acknowledgements

I thank the Vilho, Yrjö and Kalle Väisälä Foundation of the Finnish Academy of Science and Letters for the financial support.

I thank Professor Esko Valkeila for his prudent guidance during the final stages of the project. I thank Professor Olavi Nevanlinna for always providing the appropriate direction. I thank other staff at the Institute of Mathematics for their help in getting things done.

I thank my co-authors for their contributions to the articles, especially Tuomas Pyykkönen for the correctness and Dr. Jussi Keppo for the inspiration. I thank Dr. Teemu Pennanen and the group at the Helsinki School of Economics for the always rich discussions. I thank my employer Fortum for giving me the opportunity to pursue these academic interests and my colleagues for an enjoyable work environment.

I thank my family and friends for their backing, understanding, and love. You have been invaluable.

# Publications

(I) I. Vehviläinen and T. Pyykkönen. Stochastic factor model for electricity spot price - the case of the Nordic market. To appear in *Energy Economics*, 20 pages.

(II) N. Audet, P. Heiskanen, J. Keppo, and I. Vehviläinen. Modeling electricity forward curve dynamics in the Nordic market. In D. W. Bunn, editor, *Modelling Prices in Competitive Electricity Markets*, pages 251–265, Wiley, 2004.

(III) I. Vehviläinen. Basics of electricity derivative pricing in competitive markets. *Applied Mathematical Finance*, 9(1):45–60, 2002.

(IV) I. Vehviläinen and J. Keppo. Managing electricity market price risk. *European Journal of Operational Research*, 145(1):136–147, 2003.

In [I] the author is responsible for the main part of the analysis, model implementation, and writing. The model was developed in collaboration with others. In [II] the author contributes to the model of the price dynamics, and its application to the Nordic market and price forecasting. In [III], the author's independent research is reported, while, in [IV], the author is responsible for the main part of the analysis, model implementation, and writing, whereas the model is based on the idea of his co-author.

# Contents

# 1 Introduction

Financial markets transfer financial resources, such as capital, equity, and credit, between various areas of the economy. Investors participating in financial markets seek to benefit from transactions taking place. The monetary value of financial securities provides a quantitative base for mathematical analysis of the markets. Mathematical finance attempts to provide mathematical explanations for the behaviour of financial markets.

Globalisation has increased competitive pressures for the local economies. This has lead to a quest for more efficient market structures and for privatisation and deregulation in many fields. Deregulation of the electricity industry has introduced free competition to electricity generation and sales businesses. The intent has been to benefit the industries and other consumers by creating an efficient competitive market and thus to lower the cost of electricity. The natural monopolies of physical electricity transmission and distribution remain closely regulated.

In the deregulated Nordic electricity market, generators, sales companies, and large end users trade physical electricity and financial electricity derivative instruments in an electricity exchange. The exchange sets the physical electricity price for the next day, the *spot price*, with an equilibrium model that matches the supply and demand curves of market participants. Supply and demand must be in balance at each instance separately to maintain the technical functionality and operational reliability of the power system. Technically, it is impossible to store electricity in large quantities in an economically feasible manner. Therefore, variations in electricity generation or in electricity demand cause considerable price uncertainty, i.e. high *volatility*, in spot electricity prices. Market participants use financial derivative instruments to manage the uncertainties of future spot prices or to speculate upon the uncertainty. Functionality and practises of competitive electricity markets are in many ways similar to financial and other commodity markets.

In the language of mathematics, a probability space $(\Omega, \mathcal{F}, P)$ models the uncertainty in the market. Here $\Omega$ is a set of possible outcomes for random events, $\mathcal{F}$ is a $\sigma$-algebra on $\Omega$, and $P$ is a probability measure defined on $\mathcal{F}$. A random variable $X(\omega)$ is a $\mathcal{F}$-measurable function $X(\cdot) : \Omega \to \mathcal{S}$, where $(\mathcal{S}, \mathscr{S})$ is a measurable state space that describes the values of random outcomes in a topological space $\mathcal{S}$ and $\sigma$-algebra $\mathscr{S}$ on $\mathcal{S}$. A *stochastic process* is a collection of random variables that occur at each $t \in \mathcal{T}$, where $\mathcal{T}$ is a set of times in an interval $\mathcal{T} \in [0, T]$. A stochastic process is called a *discrete time* process if the set $\mathcal{T}$ is discrete and a *continuous time* process if $\mathcal{T} = [0, T]$. A nondecreasing family $\{\mathcal{F}_t\}_{t \geq 0}$ of sub-$\sigma$-fields of $\mathcal{F}$ defines a *filtration* for which $\mathcal{F}_s \subseteq \mathcal{F}_t \subseteq \mathcal{F}$ for $0 \leq s < t < T$. A stochastic process $X(t, \omega)$ is *adapted* to $\mathcal{F}_t$ if $X(t, \omega) : [0, T] \times \Omega \to \mathcal{S}$ is $\mathcal{F}_t$-measurable for each $t$.

In the financial context, an $\mathbb{R}^{n+1}$-valued $\mathcal{F}_t$-adapted stochastic process $S(t, \omega)$ models market price movements, i.e. $\mathcal{S} := \mathbb{R}^{n+1}$ and $\mathscr{S} := \mathscr{B}(\mathbb{R}^{n+1})$, the $\sigma$-field of Borel sets. The process $S(t, \omega)$ is referred to as a *market*. $S_i(t, \omega)$ denotes the price of asset $i$. A filtration $\mathcal{F}_t$ contains the market information available at time $t$. A *contingent claim* is a security that gives its owner a stochastic $\mathcal{F}_t$-adapted cash-flow $F(t, \omega)$. A contingent claim is a *derivative* of an *underlying asset* $S(t, \omega)$ if $F(t, \omega)$ is a measurable function of $S(t, \omega)$.

Some key questions in mathematical finance are the modelling of market price processes, derivative instrument pricing, and the selection of a portfolio of securities in a certain optimal sense. These same questions are of interest in competitive electricity markets.

# 2 Mathematical finance

## 2.1 Stochastic modelling

In finance, a general assumption is that price movements are random. Bachelier (1900) was the first to formulate stock price movements with a stochastic process. More precisely, he modelled stock price movements with a Brownian motion $B(t, \omega)$. The movements of a Brownian motion are random and its state is normally distributed at any given time. The increments of a Brownian motion are independently distributed. Finally, there is a version of Brownian motion for which the mapping $t \to B(t, \omega)$ is continuous for almost all $\omega$[1].

An extensive period of empirical and theoretical research on stock prices followed the work of Bachelier, but mostly in the latter half of the 1900s. Muth (1961) developed a general theory of rational expectations which was complemented by Lucas (1972, 1976). The theory of rational expectations states that people form expectations as rationally as they can on the basis of available information. Samuelson (1965a) and Mandelbrot (1966) explained stock price movements in a similar manner. Their work, together with the empirical and theoretical research of Fama (1965, 1970), led to the development of the *efficient market hypothesis*. According to this hypothesis, stock prices already incorporate all available information. Otherwise, predictable price movements would generate possibilities for speculators to gain risk-free profits. In efficient markets such speculators always exist and they always take advantage of the presented opportunities, thus causing the opportunities to disappear.

Prices following a Brownian motion can become negative. To overcome this shortcoming, Samuelson (1965b) formulated the price movements of a single stock as a geometric Brownian motion. For the price of a stock $i$ following a geometric Brownian motion

$$S_i(t, \omega) = e^{\sigma_i B_i(t,\omega) + \delta_i t},$$

where $B_i(t, \omega)$ is a one-dimensional Brownian motion and $\sigma_i \in \mathbb{R}$ and $\delta_i \in \mathbb{R}$ are parameters describing stock $i$. A stochastic process that follows a

[1]Karatzas and Shreve (1988), for example, gave a more thorough treatment of these results by Wiener.

geometric Brownian motion is log-normally distributed. Geometric Brownian motion is still the most widely used process for stock price modelling, due to its simplicity.

More generally, a geometric Brownian motion is an example of an *Itô diffusion process*. For an Itô diffusion process, the underlying price movements in $(n+1)$-dimensions follow

$$\mathrm{d}S(t,\omega) = \mu(t, S(t,\omega))\mathrm{d}t + \sigma(t, S(t,\omega))\mathrm{d}B(t,\omega), \tag{2.1}$$

where $B(t,\omega)$ is an $(n+1)$-dimensional Brownian motion, $\mu(t,S) : [0,T] \times \mathbb{R}^{n+1} \to \mathbb{R}^{n+1}$ and $\sigma(t,S) : [0,T] \times \mathbb{R}^{n+1} \to \mathbb{R}^{(n+1)\times(n+1)}$. For (2.1) to have a solution, it is typically assumed that the functions $\mu(t, S(t,\omega))$ and $\sigma(t, S(t,\omega))$ satisfy Lipschitz continuity and growth conditions

$$\begin{aligned} \|\mu(t,x) - \mu(t,y)\| + \|\sigma(t,x) - \sigma(t,y)\| &\leq K\|x-y\|, \\ \|\mu(t,x)\|^2 + \|\sigma(t,x)\|^2 &\leq K^2(1 + \|x\|^2), \end{aligned}$$

for some constant $K \in \mathbb{R}$ and for every $t \in [0,T]$, $x \in \mathbb{R}^{n+1}$, and $y \in \mathbb{R}^{n+1}$. Itô diffusion processes cover a wide variety of price movements. For example, Föllmer and Schweizer (1993) analysed economic theory behind some stock price diffusion processes. Economic analysis and explanations are important to the transparency and credibility of models in applications. Merton (1975) constructed various stochastic processes based on an economic growth model. Ross (1976) provided economic justification for asset price processes by constructing a simple factor model for the random asset returns. Bielecki and Pliska (1999) modelled underlying economic factors directly and used these factors to construct market-price processes. Examples of their economic factors are stock dividend yields, short-term interest rates, and inflation rates.

Itô diffusion processes have continuous paths that are unable to capture peaks or jumps in prices. Merton (1976) and Cox and Ross (1976) introduced jump diffusion processes that have a random component to represent possible jumps in prices. Duffie et al. (2000) presented a general affine jump diffusion process

$$\mathrm{d}S(t,\omega) = \mu(t, S(t))\mathrm{d}t + \sigma(t, S(t))\mathrm{d}B(t,\omega) + \mathrm{d}q(t, S(t)), \tag{2.2}$$

where $q(t, S(t))$ follows a pure jump process with intensity $\lambda(t)$. Mathematically, it is possible to generalise (2.2) and construct more versatile models. For example, Carr and Wu (2004) considered a general model based on a stochastic time-changed Lévy process. The economic motivation of such generalisations is necessary to connect the models to the observed price movements.

In commodity markets, there are typically two ways of trading a commodity. The commodity can be traded at spot price now. Alternatively, the delivery of the commodity can occur at some future time point and the price can be agreed upon with a forward contract now. Commodity prices are influenced by non-financial factors, such as storage costs, transportation, and utility of commodity consumption that vary over time. Therefore, forward prices do not have an explicit dependency on spot prices. Commodity price modelling needs to consider both spot prices and *forward price dynamics*. Interest rate markets share a similar problem setting although for different reasons. Commodity price modelling has followed the developments of stock price and interest rate modelling.

Interest rate markets give comparative values of money at different future times as seen by investors today. The future value of money is dependent on inflation and other economic fundamentals that are stochastic. Vasicek (1977) presented a *mean-reverting* process for the short-term interest rate $\rho(t,\omega)$. Mean-reversion implies that interest rates fluctuate around some mean interest level. In Vasicek's model, the short-term interest rate followed an Itô diffusion process

$$\mathrm{d}\rho(t,\omega) = a(b - \rho(t,\omega))\mathrm{d}t + \sigma \mathrm{d}B(t,\omega), \tag{2.3}$$

where $b \in \mathbb{R}$ gives the mean interest level, $a \in \mathbb{R}$ the rate of change toward $b$, and $\sigma \in \mathbb{R}$ the local volatility. The process (2.3) is also known as the mean-reverting Ornstein-Uhlenbeck process. The theoretical rationale behind Vasicek's model was that if interest rates become too high, the resulting economic slow-down will eventually bring them down. On the other hand, if the interest rates are too low, economic activity increases and interest rates will rise. Cox et al. (1985) and Hull and White (1990a) presented further models for the short-term interest rate.

Heath, Jarrow, and Morton (1992) modelled the interest rate forward curve dynamics. Their model allowed stochastic movements at different points in the forward curve. An increase in the number of factors that approximate the forward curve increases the explanatory power of the model. Litterman and Scheinkman (1991) concluded that three factors explain most of the observed interest rate movements. One factor explains the overall shifts in the interest rate level, second changes in the slope of the interest rate curve, and third variations in the curvature. Björk and Landén (2002) presented several models and applications for interest rate forward curve dynamics.

In commodity markets, economic theory supports the development of commodity price processes. Working (1949) developed the theory of the influence of storage on inter-temporal price dynamics. Mandelbrot (1966) used the efficient market hypothesis to model commodity prices like stock prices. Kawai (1983) considered the implications of the theory of rational expectations without storage possibilities, and Williams and Wright (1991) and Deaton and Laroque (1996) studied storable commodities.

Brennan and Schwartz (1985), among others, used geometric Brownian motion to model commodity spot price movements. Geometric Brownian motion gives a first approximation of random commodity price movements just as it does in financial markets. For instance, Smith and McCardle (1998) modelled oil prices and Davis (2001) studied derivative pricing on accumulated weather indexes based on geometric Brownian motion. Duffie and Gray

(1995) and Schwartz (1997) presented and analysed more general diffusion processes to represent commodity prices. Their processes incorporated some of the observed commodity price characteristics, such as seasonality, mean-reversion, and serial correlation, better than geometric Brownian motion. Benth (2003) considered even more versatile fractional Brownian motions to price weather derivatives. Modelling results from one market are not necessarily applicable to other markets because commodity price characteristics are market dependent.

There have been several attempts at modelling electricity spot prices in competitive markets. The non-storability of electricity does not support the arguments about the price formation as a continuous stochastic process. However, that does not prevent modelling the random characteristics of prices with stochastic processes. The available literature has two main branches: statistical models and fundamental models. Statistical, or econometric, models follow the finance tradition of modelling directly the stochastic processes that represent prices. Fundamental, or structural, models build the price processes based on equilibrium models for the electricity market. Article [I] of this thesis presents a fundamentally motivated spot price model that combines statistical processes for the factors affecting spot prices with an approximative market model.

In the statistical approaches, modelling concentrates on the price process form and parameters, such as functions $\mu(t, \omega), \sigma(t, \omega), \lambda(t, \omega)$ of the process in equation (2.2). After process form selection, available historical market data provide estimates for the parameters so that the model matches the historical prices. In U.S. markets, Deng (2000) presented mean-reverting jump diffusion spot price processes with two jump types. He modelled price volatility with either a deterministic model, a model with two regimes between which the price jumps, or with a stochastic model. Weron and Przybyłowicz (2000) analysed the mean-reverting property in detail and concluded that, statistically, at least Californian electricity market prices are mean-reverting. Davison et al. (2002) modelled the average power demand and generation capacity. In their model, the electricity price switches randomly between two price regimes on the basis of the ratio between demand and capacity. In the Nordic electricity market, Lucia and Schwartz (2002) considered several diffusion processes. Their models were extensions of Vasicek's model for interest rates that included a deterministic time-dependent component. Simonsen (2003) analysed hourly Nordic spot prices and concluded that prices are mean-reverting. Huisman and Mahieu (2003) presented a regime-switching model where price spikes were separated from mean-reverting prices. Weron et al. (2004a) studied a jump diffusion model and a regime-switching model. Weron et al. (2004b) considered the explanations behind several models and ended up recommending a mean-reverting jump diffusion model.

The construction of statistical models is easy but no rigorous economic motivation for the parameters has yet been given in electricity markets. In comparison to financial markets, electricity markets lack the long historical time series that would allow process parameter estimation. The continuous structural and regulatory changes[2] in the markets have a major effect on the prices. The historical estimates are not necessarily valid in the future, and the influence of market changes to parameter values can be difficult to estimate.

In the fundamental approaches, a model for the supply–demand balance determines electricity prices. Botnen et al. (1992) and Haugstad and Rismark (1998) presented a model for the minimisation of the marginal generation cost of the whole generation system in the Nordic market against consumption and transmission constraints. They assumed that, in a competitive market, spot price equals the marginal generation cost thus obtained. Johnsen (2001) presented a supply–demand model for the hydro-dominant Norwegian electricity market from a time before the common Nordic market had started. He used hydro inflow, snow, and temperature conditions to explain spot price formation. Many approaches supplement fundamental models with statistical processes. Skantze and Ilic (2001) considered a fundamental model for the electricity price dynamics that incorporated the seasonality of prices, stochastic supply outages, and mean revision. Barlow (2002) used a mean-reverting process for the demand and a fixed supply function to end up with a mean-reverting process for the spot price. Burger et al. (2004) created a model that included a stochastic load process and used statistical processes to describe the remaining errors. Models that include fundamental factors are more tractable than statistical models. Economic reasoning can be used to deduct the properties of the factors. The special characteristics of electricity prices and changing market conditions are better captured with fundamental models than with pure statistical models. On the other hand, fundamental models require comprehensive data sets that are laborious to maintain.

Visudhiphan and Ilic (1999) attempted to take the strategic behaviour of market participants into account. They modelled the dynamic bidding strategies of generators and assumed that the demand is given. They argued that such a model leads to a dynamic equilibrium that reflects spot prices. Hobbs et al. (2000) modelled the price formation of one single time period with a game-theoretic approach. Anderson and Philpott (2002) considered the formation of supply-function equilibrium, while Hinz (2003) studied equilibrium prices and optimal bidding strategies for electricity producers under two different electricity auction frameworks. These economic and game theoretic models can give insights to the market dynamics but their capability to explain observed price levels and dynamics has been weak.

Koekebakker and Ollmar (2001) considered the electricity forward price dynamics in the Nordic electricity market using the Heath-Jarrow-Morton framework. Their conclusion was that two stochastic factors are unable to explain the forward curve dynamics as well as two factors in interest rate markets. Lucia and Schwartz (2002) used one-factor and two-factor models to explain forward prices. They concluded that their models perform worse than

[2]For example, the planned fifth nuclear power plant in Finland will change supply conditions and EU wide emission trading scheme will change the marginal costs of carbon emitting power plants.

similar models in other commodity markets. Benth et al. (2003) extended a spot price model presented by Schwartz (1997) to incorporate jumps and time varying parameters, and derived model-based forward prices. They used risk premiums to explain the difference between model prices and observed market prices. They applied the model to oil markets and the Nordic electricity market. Again, the model was able to explain price developments in the oil markets better than in the electricity markets. The modelling of electricity forward price dynamics is an active research topic to which article [II] of this thesis contributes.

## 2.2 Derivative pricing

Bachelier (1900) was the first to consider mathematical pricing of a financial derivative in connection with his stock price model, i.e. the Brownian motion. Since then, financial markets have adopted a large number of derivatives that are actively traded. The most common derivatives are forward and futures contracts and options. A *forward contract* is an agreement to buy or sell an asset at a certain future time at a certain price. The cash flow from a forward contract, or the *payoff*, is given by

$$F(S(T,\omega)) := S(T,\omega) - K,$$

where $S(T,\omega)$ is the price of the underlying asset at time $T$ and $K \in \mathbb{R}$ is the fixed price agreed upon. A futures contract has similar payoff but slightly different payment terms. Two basic options are a *call option* and a *put option*. The owner of a call option has the right to buy the underlying asset at a certain price. The owner of a put option has the right to sell the underlying asset at a certain price. *European* exercise rules mean that the right to exercise an option is restricted to a fixed time $T$. With *American* exercise rules, the exercise can occur at any time in an interval $[0,T]$. As an example, a European call option has a payoff

$$F(S(T,\omega)) := (S(T,\omega) - X)^+,$$

where $T$ is the fixed exercise time and $X \in \mathbb{R}$ is the fixed exercise price.

In their seminal paper, Black and Scholes (1973) considered the pricing of European options on stock price. They assumed that stock prices follow a random walk, the stock price distribution at any time is log-normal, the stock does not pay dividends, and interest rates are constant and deterministic, denoted by $\rho \in \mathbb{R}$. A stochastic process that describes these stock price characteristics is a geometric Brownian motion with some growth rate $\mu$ and volatility $\sigma$. Given such circumstances, Black and Scholes showed that the theoretical price of the option $f(t,S(t,\omega))$ follows a partial differential equation

$$\frac{\partial f(t,S)}{\partial t} + \rho S(t,\omega)\frac{\partial f(t,S)}{\partial S} + \frac{1}{2}\sigma^2 S^2(t,\omega)\frac{\partial^2 f(t,S)}{\partial S^2} = \rho f(t,S), \qquad (2.4)$$

before the exercise time. The option payoff at $T$ gives a boundary condition for the partial differential equation. For a European call option, the boundary condition is

$$f(T, S(T, \omega)) = F(S(T, \omega)) = (S(T, \omega) - X)^{+}. \tag{2.5}$$

The solution to the partial differential equation (2.4) together with the boundary condition (2.5) gives the pricing formula for a European call option

$$f(t, S(t, \omega)) = S(t, \omega)\Phi(d_1) - e^{-\rho(T-t)} X \Phi(d_2), \tag{2.6}$$

where $\Phi(\cdot) : \mathbb{R} \to [0, 1]$ denotes the cumulative distribution function for a normally distributed random variable with a mean of 0 and a standard deviation of 1, and

$$\begin{aligned} d_1 &= \frac{\ln S(t, \omega)/X + (\rho + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}, \\ d_2 &= d_1 - \sigma\sqrt{T - t}. \end{aligned}$$

The basis for the Black and Scholes pricing formula is the no-arbitrage argument and the use of continuous trading to replicate option payoffs.

More formally, a *trading strategy* is a $\mathcal{F}_t$-adapted stochastic process $\pi(t, \omega)$. The values $\pi_i(t, \omega), i = 0, \dots, n$, describe the portfolio holdings at time $t$, i.e. the number of units of asset $i$ held at time $t$. A *self-financing* trading strategy requires no investment after the initial cost. A self-financing trading strategy is *admissible* if the associated portfolio wealth, $\pi^{\mathrm{T}}(t)S(t, \omega)$, has a finite lower bound. This lower bound limits the amount of money that an investor can borrow. An *arbitrage* is a self-financing trading strategy that has zero initial cost, a non-negative end wealth $P$-almost surely, and a positive end wealth with positive probability. A self-financing trading strategy *replicates* a contingent claim if the payoff from the trading strategy is equal to the payoff from the contingent claim, $P$-almost surely. Black and Scholes derived their partial differential equation (2.4) by constructing a replicating trading strategy for the option payoff. The cost of starting the replication strategy gives the theoretical option price as given by the pricing formula (2.6). If the real option price is lower (higher) than the initial cost of replication, it is theoretically possible to buy (sell) the option, follow the replicating strategy, and be left with a certain profit at the exercise time. A similar no-arbitrage argument for a forward contract with delivery time $T$ yields the theoretical arbitrage-free forward price as

$$f(t, S(t, \omega); T) = e^{\rho(T-t)} S(t, \omega),$$

provided that the interest rates are deterministic and constant, denoted by $\rho \in \mathbb{R}$, and that the underlying asset is tradable.

Research and practical developments on derivative instruments have built on the work by Black and Scholes. Merton (1973) relaxed the assumption that the stock does not pay dividends and presented the results of Black and

Scholes more coherently. The basic financial market model has developed since then. Based on Ross (1976), a standard assumption is that the market is arbitrage-free. Another common assumption is that the market has an asset, for example cash, whose value is positive $P$-almost surely. The positive asset makes it possible to construct a *normalised* price process by dividing the original price process $S(t,\omega)$ by the asset that is always positive. The value of money held in a risk-free interest bearing bank account is a typical denominator. In the case of constant and deterministic interest rates, the interests accumulate by

$$S_0(t) = e^{\rho t} S_0(0),$$

where $S_0(t)$ denotes the nominal amount of money and $\rho$ gives the interest rate level. Then, the denominator is

$$\xi(t) := 1/S_0(t) = e^{-\rho t},$$

and the normalised price process is

$$\tilde{S}(t,\omega) := \xi(t) S(t,\omega).$$

In general $\xi(t)$ can be stochastic; for example, if the interest rates are stochastic.

A replicating trading strategy is independent of investors' risk preferences. Cox and Ross (1976) constructed *risk-neutral* probabilities under which the theoretical option price is equal to the expectation of the option payoff. A risk-neutral probability measure is closely linked with the concept of an equivalent martingale measure. A measure $Q$ is *equivalent* to $P$ if

$$P(F) > 0 \text{ if and only if } Q(F) > 0, \forall F \in \mathcal{F}.$$

For an absolutely integrable process $X(t,\omega)$,

$$\mathrm{E}^Q[|X(t,\omega)|] < \infty, \quad \forall t \geq 0.$$

An absolutely integrable $\mathcal{F}_t$-adapted process $X(t,\omega)$ is a *martingale* with respect to a measure $Q$ if

$$\mathrm{E}^Q[X(t,\omega)|\mathcal{F}_s] = X(s,\omega), \quad \forall s \leq t,$$

where $\mathcal{F}_s$ is the filtration that gives the information at time $s$. Harrison and Kreps (1979) created a unified functional analytic framework for derivative instrument pricing. They showed that a market $S(t,\omega)$ has no admissible arbitrage opportunities if and only if an equivalent martingale measure exists[3]. Moreover, they showed that the theoretical price of any contingent claim

[3]Actually, in continuous-time models and if $T$ is not finite, Delbaen and Schachermayer (1994, 1998) showed that the existence of a martingale measure requires a slightly stronger condition: *no free lunches with vanishing risk.*

$f(t, S(t, \omega))$ can be stated as a stochastic integral of a replicating trading strategy, $\pi(t, \omega)$, i.e.

$$\xi(T, \omega) f(T, S(T, \omega)) = f(0, S(0, \omega)) + \int_0^T \pi(t, \omega) \mathrm{d}\tilde{S}(t, \omega), \qquad (2.7)$$

where $\xi(t, \omega)$ is the normalisation function and $\tilde{S}(t, \omega)$ the normalised market. An equivalent martingale measure $Q$ for $\tilde{S}(t, \omega)$ exists in arbitrage-free markets. Taking expectations of both sides of (2.7) with respect to $Q$, yields the theoretical price of the contingent claim as

$$f(0, S(0, \omega)) = \mathrm{E}^Q[\xi(T, \omega) f(T, S(T, \omega)) | \mathcal{F}_0]. \qquad (2.8)$$

A market $S(t, \omega)$ is *complete* if each bounded contingent claim can be replicated. As an example, the Black and Scholes market is complete. If an equivalent martingale measure $Q$ exists, Harrison and Pliska (1981) showed that $Q$ is unique if and only if the market is complete. If markets are incomplete, it follows that an equivalent martingale measure $Q$ is not unique. In incomplete markets, the risk-neutral pricing theory does not guarantee unique theoretical prices for derivatives.

Market incompleteness arises due to real-world market *frictions*. Some market frictions are transaction costs, constraints on trading, taxation effects, and different rates for borrowing and lending money. Stochastic modelling of the market can also cause incompleteness; for example, a market following a jump-diffusion process (2.2) is incomplete in general. The advances in derivative pricing theory in incomplete markets have focused on the inclusion of some specific friction. Karatzas (1989), Sundaresan (2000), and Cvitanic (2001) presented some of these developments. The value of these theoretical advances is limited outside their respective areas. King (2002) considered many simultaneous market frictions. He presented an arbitrage-free interval for European contingent claim prices by using stochastic programming. Kallio and Ziemba (2003) presented arbitrage-free contingent claim pricing in incomplete markets that is based on simple optimisation theory. These developments are promising in their simplicity and generality.

Despite the theoretical advances in derivative pricing, quantitative pricing remains challenging. In certain simple instances, either the partial differential equation or the integral form that describes the theoretical derivative prices can be solved analytically. A typical requirement is that the market is complete; this gives rise to a unique solution to the martingale pricing problem if an equivalent martingale measure exists. Also, if a derivative is of the European type, its payoff gives a boundary condition for the pricing partial differential equation. If the exercise occurs according to the American exercise rules, the theoretical results are less satisfying. Jacka (1991) presented some results on integral equation representation for the price of an American put option. Karatzas and Shreve (1998, p. 54–79) described the pricing problem of the American contingent claims in more detail.

Numerical methods generate quantitative price estimates if there are no analytical solutions to derivative prices. Based on special market or derivative characteristics, it is sometimes possible to approximate the theoretical derivative prices analytically. Johnson (1983) and MacMillan (1986) studied the pricing of American contingent claims for which there are no analytical pricing formulae. Given the characteristic function of the underlying probability distribution, Carr and Madan (1998) and Lee (2004) calculated theoretical derivative prices using Fourier transformations. Brennan and Schwartz (1978), Courtadon (1982), and Hull and White (1990b) used finite difference methods to numerically solve the partial differential equation associated with the theoretical contingent claim price. These methods are highly dependent on market characteristics and derivative properties.

One method of quantitative pricing is to integrate numerically the expectation present in the pricing problem. In general, the expectation of random variable $X : \Omega \to \mathbb{R}^{n+1}$ with respect to a distribution $P$ can be approximated with

$$E^P[X] := \int_{\mathbb{R}^{n+1}} X(\omega)P(\mathrm{d}\omega) \approx \frac{1}{p}\sum_{k=1}^{p} c^k X(\omega^k),$$

where the choice of points $\{\omega^k\}_{k=1,\ldots,p} \in \Omega$ and weights $\{c^k\}_{k=1,\ldots,p} \in \mathbb{R}$ should lead to increasing accuracy as $p \to \infty$. In Monte Carlo simulation methods, this approximation is random. Random variables $\{\omega^k\}_{k=1,\ldots,p}$ are drawn according to the distribution $P$ and equal weight is assigned to each sample. Boyle (1977) introduced Monte Carlo methods to solving financial problems. Boyle et al. (1997) provided several technical improvements of Monte Carlo methods in pricing of American options, for example. Lattice and tree methods determine the possible price movements explicitly. The point locations and weight sizes are fixed according to some theoretical or heuristic arguments. Cox et al. (1979) presented the theory of risk-neutral option pricing in discrete time with a simple binomial model for the price movements. King (2002) formulated the option price evaluation as a stochastic programming problem and allowed more general tree models for uncertain factors.

Commodity derivatives are market specific and a limited number of common pricing results exists. Black (1976) derived the theoretical price of a European option on commodity forward prices by assuming that the forward prices follow a log-normal process. Brennan and Schwartz (1985) presented pricing formula for commodity forward contracts if there were storage possibilities for the commodity. Gibson and Schwartz (1990) studied the pricing of contingent claims in oil markets under a particular price process. Miltersen and Schwartz (1998) extended Black's model to specific interest rate and forward price structures. As in electricity markets, non-storable underlying assets exist in some livestock markets. Tomek and Peterson (2001) studied derivative pricing in agriculture following the finance traditions.

Commodity derivatives or physical assets that are operated in the market have often complex payoffs. Brennan and Schwartz (1985) studied the valuation of natural resources that are used with flexible decisions. Dixit and

Pindyck (1994) analysed the related concept of *real options* and gave several application areas. Real option theory considers the flexibility embedded in specific investment opportunities. Complex derivatives in commodity markets and real investment opportunities share similar characteristics. On gas markets, Thompson (1995) used a lattice-based framework to analyse the theoretical price of a *swing option*[4]. Commodity pricing is often based on the assumptions on the particular price process form and therefore pricing results are not as general as in financial markets.

In competitive electricity markets, Kaye et al. (1990) presented early ideas on forward contract pricing. Gedra (1994) studied pricing of electricity forward contracts and options on spot price with the Black and Scholes approach. Similarly, Ghosh and Ramesh (1997) priced spot electricity forwards and options directly as in financial markets. However, because physical electricity is not a tradable asset, there are no theoretical grounds to motivate replicating portfolios for spot electricity derivatives as in the Black and Scholes model. As a consequence, spot electricity markets are incomplete. Article [III] of this thesis contributes to the general theory and basic properties of electricity derivative pricing.

Eydeland and Geman (1999) considered a generation-based fundamental model for the theoretical prices of forward contracts. Bessembinder and Lemmon (2002) presented an equilibrium model for the spot prices and used the model to price forward contracts. These models rely on some subjective views on spot price movements. A general result for the pricing of electricity forward contracts has not yet been established.

Similarly, pricing results on more complicated electricity derivatives are based on subjective price process views. Bjorgan et al. (2000) studied swing option pricing in electricity markets. They obtained theoretical swing option prices with a particular spot price process by using stochastic dynamic programming. Lari-Lavassani et al. (2001) valued swing options under certain price processes by creating a set of scenario trees to represent the optionality of the contract. Mo and Gjelsvik (2002) presented an optimal strategy for the holder of a swing option under their price model but did not consider the pricing problem as such. Tseng and Barz (2002) took a real options approach and considered the valuation of generation assets with a Monte Carlo application of dynamic programming.

Market incompleteness restricts theoretical pricing of derivatives on electricity spot prices. If all future time points can be traded with forward contracts, then the resulting forward contract market is complete. On this kind of forward market, complicated derivative pricing is possible. For instance, Deng et al. (2001) introduced pricing models for generation assets, while Keppo and Räsänen (1999) studied end consumer sales. However, it is a strong requirement to have a tradable forward contract for each time period. It is yet to be shown that the assumption of a completed forward market is feasible in electricity markets.

[4]See, for example, article [III] for a definition.

## 2.3 Investment theory

An investor in the market faces alternative investment opportunities that result in uncertain outcomes. Based on the investor's preferences, he/she ought to invest the available capital in an "optimal" manner. The main questions are how to characterise investors' preferences and the optimality of a portfolio, how to model the uncertainties involved, and how to make the optimal selection. Article [IV] of this thesis studies these questions in the competitive electricity markets.

The theory of optimal portfolio selection originates from the work by Markowitz (1952a, 1959) who considered the choice between several risky assets, such as stocks in the stock market. The Markowitz model is set in a simple one-period market $\mathcal{T} = \{0, T\}$. An investor selects a portfolio $\pi \in \mathbb{R}^{n+1}$ at the beginning of the time period, i.e. invests $\pi_i$ in an asset $i$ at a price $S_i(0)$, and observes the market prices $S(T, \omega) \in \mathbb{R}^{n+1}$ at the end of the period. The simple asset returns $r \in \mathbb{R}^{n+1}$ over time are

$$r_i := \frac{S_i(T, \omega) - S_i(0)}{S_i(0)}, \quad \forall i = 1, \dots, n+1.$$

The return of the whole portfolio at the end of the time period is $r^{\mathrm{T}}\pi$. Markowitz assumed that there are estimates for the expected asset returns $\bar{r}$ and the covariances of assets $V$ over the time $[0, T]$. The investor targets some level for the expected return, $r^* \in \mathbb{R}$. The optimal portfolio in the sense of Markowitz is given by the solution of

$$\begin{aligned} \underset{\pi}{\mathit{minimise}} \quad & \frac{1}{2}\pi^{\mathrm{T}} V \pi \\ \mathit{subject\ to} \quad & \bar{r}^{\mathrm{T}}\pi = r^*, \\ & \sum_{i=1}^{n+1} \pi_i = 1. \end{aligned}$$

Such selection results in an estimated variance minimising portfolio that has an estimated expected return $r^*$. The optimal portfolio selection is dependent on the estimates for returns and covariances.

Markowitz's theory focused on the choice between several risky assets. Tobin (1958) included the possibility of holding capital in a risk-free asset, such as cash, instead of investing everything in the stock market. He argued that a rational investor holds some capital in risk-free money and some in a single portfolio of risky assets. The risk attitudes of investors determine the division between risk-free investment and risky investment.

The optimal portfolio selection theory of Markowitz and Tobin introduced risk to investment strategy considerations. The attitude toward risk vs. return defines what is optimal for each investor. Von Neumann and Morgenstern (1944) created an axiomatisation of *risk preferences*. They showed that if the investor can always arrange presented alternatives in the order of his/her preference, then a *utility* function can be associated with those preferences. It is customary to consider the utility of wealth $U(W) : \mathbb{R} \to \mathbb{R}$ in finance; as in article [IV]. A utility function is usually assumed to be a twice continuously differentiable increasing function of wealth. Portfolio *wealth* is given by

$$W(\pi(t), S(t,\omega)) := \pi^{\mathrm{T}}(t) S(t,\omega),$$

where $\pi(t)$ gives the portfolio holdings and $S(t,\omega)$ gives the market prices for the assets.

Friedman and Savage (1948) studied utility functions and their connection with the implied risk preferences. They categorised investors as *risk-averse*, *risk-neutral*, or *risk-seeking*. Given two investment opportunities with equal expected return, risk-averse investors choose the one with lesser risk. Risk-neutral investors are indifferent between a risky and a certain investment if the investments have equal expected return. Risk-seeking investors prefer the risky investment more than the certain investment. Mathematically, the risk preferences translate to the properties of utility functions. The risk-averse investor's utility function is strictly concave, while the risk-neutral investor's is linear and the risk-seeking investor's strictly convex. A general consensus after the work by Markowitz (1952a,b) is that most investors are risk-averse. For a risk-averse investor, an increase in wealth from modest starting wealth gives a greater increase in utility than an equal increase starting from a greater degree of wealth.

Research after Markowitz and Tobin has combined optimal portfolio selection and utility function theories. Hanoch and Levy (1969) generalised Markowitz's mean–variance setting to any von Neumann and Morgenstern utility function. Pratt (1964) and Arrow (1965) made characterisations of utility functions in terms of how large a risk premium a risk-averse investor with some specific utility function would like to receive in comparison to a risk-neutral investor. Kallberg and Ziemba (1983) studied the properties of utility functions in optimal portfolio selection problems. They concluded that the selection of a utility function to match certain risk preferences can be made on the basis of a risk characterisation similar to those of Arrow and Pratt. Luenberger (1993) considered theoretical justifications for choosing a particular utility function that maximises the expected growth, while Hakansson and Ziemba (1995) considered the importance of utility function choice for the long-term growth in wealth. Rabin (2000) and Rabin and Thaler (2001) criticised the utility theory and the assumption that rational investors are risk-averse. They showed that the estimation of a concave utility function based on a decision on a certain level of wealth implies irrational choices on other levels of wealth. They suggested an alternative functional form that would capture risk preferences in a more relevant manner.

Modern optimal portfolio selection theory is based on utility maximisation over time. The static one-period optimal portfolio selection theory was elaborated to a dynamic multi-period setting by Samuelson (1969) in discrete time and by Merton (1969, 1971) in continuous time. A dynamic optimal portfolio selection framework allows portfolio adjustments either in discrete time steps or continuously. If the investment period is long, the possibility of adjusting the portfolio according to new information yields better optimisation results. Steinbach (2001) presented some more recent advances and refinements made in the modern optimal portfolio selection theory.

Companies participating in uncertain markets often have an interest in decreasing their profit uncertainty, or, in other words, in *hedging* against risks. Smith and Stulz (1985) showed that taxation effects can favour the more constant profits of a company that is hedging. Leland (1998) argued that hedging enables greater leverage, meaning the company can better optimise its capital structure. The question of hedging can be formulated as an optimal portfolio selection problem, i.e. as a *risk management* problem.

The introduction of electricity spot markets and electricity derivative instruments created a new challenge for the, till then, regulated power industry. High spot market price volatility exposes spot market participants to a high level of profit uncertainty. Hedging with spot electricity derivatives reduces profit uncertainty. The problem is to know how much to hedge and with what instruments. Kaye et al. (1990) and Amundsen and Singh (1992) considered the use of derivatives to hedge risks in spot electricity markets. Weron (2000) presented a more detailed analysis on the special characteristics of electricity markets. Bessembinder and Lemmon (2002) argued that companies operating in competitive electricity markets are likely to benefit from reducing their profit uncertainty like companies in other markets.

Bjorgan et al. (1999) presented an application of the Markowitz model for an electricity generator that sells electricity at spot price. Their optimal portfolio selection considered the generation strategy together with hedging decisions. Fleten et al. (1997, 2002) and Mo et al. (2001) based their work on the tradition of optimal hydro-power scheduling. They studied a scenario-based stochastic programming approach for solving the optimal portfolio selection problem in electricity markets so that the optimal generation decisions and hedging decisions are coordinated. Bjørkvoll et al. (2001) optimised generation and the corresponding hedging portfolio but separated the generation and hedging decisions. They argued that hedging decisions that take place with market prices do not affect the generation decisions. Existing literature on optimal portfolio selection focuses on the optimisation problem of an electricity generator.

In addition to the influence of risk on optimal portfolio selection, the quantification of risk is an interesting question in itself. Markowitz (1952a) associated risk with the variation of portfolio return. Since then, increased competition and consequent tighter margins have increased the need to analyse risks. Baumol (1963) presented an alternative risk measure that later has become the standard risk analysis tool in financial markets, the *value-at-risk* measure. For example, Jorion (1997) presented a thorough analysis on value-at-risk. Value-at-risk measures the difference between expected wealth and a level below which the wealth is at a given probability. More precisely, the value-at-risk for the wealth in a portfolio $\pi$ at a given probability $\alpha \in (0,1)$ and over a given time $\tau$ is

$$VaR(\pi;\alpha,\tau) := \mathrm{E}^P[W(\pi,S(\tau,\omega))] - \inf\{L \in \mathbb{R} | P(W(\pi,S(\tau,\omega)) < L) \geq \alpha\},$$

where the portfolio is constant over time. Value-at-risk has some unwanted characteristics. For example, value-at-risk does not indicate how low wealth can be if the probabilistic limit breaks. Artzner et al. (1999) introduced risk measures that are more suitable for optimisation problems and give more intuitive risk quantifications. Föllmer and Schied (2002) gave a detailed analysis of the properties of risk measures. The choice of a risk measure is dependent on investors' preferences, as is the choice of a utility function.

Pilipovic (1997) applied value-at-risk to electricity markets. In finance, value-at-risk measures the potential change in portfolio wealth in the short-term. Electricity portfolios that include physical assets are held over a longer time. Lemming (2004) presented a variation of value-at-risk called the *profit-at-risk* that gives more relevant risk quantifications in electricity markets. Profit-and-risk is identical in form to value-at-risk, but the time horizon is different. Value-at-risk focuses on the short-term changes in portfolio wealth, while profit-at-risk focuses on the wealth after longer time periods.

# 3 Results

## 3.1 Spot price model

Article [I] considers an efficient and economically sound fundamental electricity spot price model for the Nordic market. The model explicitly states the discrete time stochastic processes for the fundamental factors that influence spot prices. A market equilibrium model that approximates the actual Nordic market setting combines the stochastic factors to form spot prices.

The stochastic climate factors in article [I] are temperature and precipitation. Based on these, the article models hydrological inflow and snow-pack development, which affect hydro-power generation availability. The rest of the supply consists of constantly run nuclear power, partly temperature-dependent back-pressure generation, and price-driven condensing power. The demand model has a temperature-dependent component that captures the seasonality of demand and a stochastic component. Historical observations, underlying physical phenomena, and, if needed, expert opinions justify the model construction.

Mathematically, the movements of the $n$ stochastic factors, $X(t,\omega) := (X_0(t,\omega), \ldots, X_n(t,\omega))$, follow a discrete time diffusion process

$$X_i(t+\Delta t,\omega) = \mu_i(t, X(t,\omega)) + \sigma_i(t, X(t,\omega))\epsilon_i(t,\omega),\ 0 \le t \le T, 1 \le i \le n,$$

where $\mu_i(t,X)$ is the local drift of $X_i(t,\omega)$ and $\sigma_i(t,X)$ is the local volatility from Gaussian stochastic variable $\epsilon_i(t,\omega)$. Based on the fundamental factors, the approximate market model creates a competitive equilibrium for the spot price.

In contrast to market-dependent price data, a long history is available for the estimation of fundamental factor process parameters. Therefore, the estimates for the fundamental factor processes are stabler and factors are more accurately represented than statistical models for the complicated electricity spot price process. Realised market prices provide estimates for the parameters of the supply–demand approximation. Estimation from market prices reflects the marginal generation costs of the whole market and the behaviour of market participants.

The model of article [I] produces spot price distributions efficiently in computational terms when compared to previous fundamental models. The article presents the model performance with numerical results on the Nordic market. The ideal use of the model is to create mid-term spot price distributions. Better approaches exist for the short-term price forecasting. The unavailability of accurate long-term weather forecasts, among other things, limits the possibilities of making accurate long-term electricity price forecasts. However, the model is able to capture observed fundamentally-motivated market price movements.

Explicit models for the underlying stochastic factors are useful in risk management problems, like the one in article [IV]. An example is the quantification of the risk of an electricity end user with temperature-dependent demand. The end user's volume risk would be unnoticeable without a model for the demand and the explaining temperature. An additional benefit of modelling the underlying factors explicitly is the possibility of creating and analysing single hand-picked scenarios; for example, when analysing worst-case performance.

As in earlier literature, the price model gives derivative prices under the implied probability measure $P$ by assuming that pricing in the market occurs using the same measure. The advantage created by the article is the pricing possibility of derivatives that are dependent on spot price and underlying factors simultaneously. Article [I] shows how to price a complex multi-dimensional derivative on electricity spot price and temperature and demonstrates some quantitative theoretical prices.

## 3.2 Forward price dynamics

Article [II] presents a statistical model for the electricity forward price dynamics in the Nordic market. The relation between electricity spot price and electricity forward prices is more complicated than in most financial and commodity markets, like article [III] shows. Short-term supply–demand equilibrium determines the electricity spot prices, and supply and demand must be in balance at each instance separately. As a result, spot price now is not explicitly connected with the spot price at some future time point. Similarly, no explicit connections exist between the forward prices of different delivery times.

The model for the electricity forward price dynamics captures the main characteristics of the observed spot and forward price movements. Short-term changes in, for example, weather or supply conditions affect the short-term prices to a greater extent. Therefore, spot price volatility is higher than forward price volatility. Because changes in the supply–demand conditions are limited in time, forward prices for neighbouring delivery times correlate with each other more than forward prices whose delivery times are distant from each other.

A probability space $(\Omega, \mathcal{F}, P)$ models the market uncertainty. To remain tractable, the article makes some simplifications. Parametrised exponential functions model the volatility and correlation effects. The article assumes that forward prices are equal to the expected future spot prices, i.e. at time $t$, the instantaneous forward price for delivery time $\tau$, $f(t;\tau)$, is

$$f(t;\tau) = \mathrm{E}[S(\tau,\omega)|\mathcal{F}_t],$$

where $\mathcal{F}_t$ describes the current market information. This implies that forward prices converge to spot prices when the delivery time $\tau \to t$. The model assumes that the electricity spot volatility curve $\sigma(t) : [0,T] \to \mathbb{R}^+$ is deterministic and forward prices follow log-normal distributions.

The main assumption in article [III] is that the forward prices follow a stochastic differential equation

$$\mathrm{d}f(t;\tau) = f(t;\tau)e^{-\alpha(\tau-t)}\sigma(\tau)\mathrm{d}B(t,\omega;\tau) \quad \forall t \in [0,\tau], \tau \in [0,T],$$

where $\alpha > 0$ is a real constant and $B(t,\omega;\tau)$ is a Brownian motion corresponding to a forward contract with delivery time $\tau$. The correlation structure of Brownian motions with delivery times $\tau$ and $\tau^*$ is given by

$$\mathrm{d}B(t,\omega;\tau)\mathrm{d}B(t,\omega;\tau^*) = e^{-\gamma|\tau-\tau^*|},$$

where $\gamma > 0$ is a real constant.

Use of the maximum likelihood method gives estimates for model parameters with data from the Nordic electricity market. Article [II] presents three applications and numerical examples. Firstly, the article studies the application of the forward price model to electricity forward price forecasting, given an electricity spot price forecast. Coherent spot and forward price forecasts are useful in many applications, for example, in dynamical extensions to the optimal portfolio selection model in article [IV]. The model allows the calculation of time-dependent forward price volatilities that are in use in option pricing formulae for options on forward contracts. Finally, article [II] considers the ability to explain forward price curve movements with the model. For example, the use of four factors to explain price variations captures roughly three quarters of the uncertainty for a one-year time period. The results of the article confirm similar earlier results as to the difficulty of modelling electricity forward price dynamics when compared to other markets.

## 3.3 Electricity derivative pricing

Article [III] considers the pricing problems for the most common derivatives in the Nordic market. The article introduces a mathematical framework for the assets and instruments in the financial tradition. The underlying market $S(t,\omega)$ is modelled in a probability space $(\Omega, \mathcal{F}, P)$.

The article associates the non-storability of electricity with the non-tradability of spot electricity. It is impossible to buy electricity from the market now, hold it over time, and sell it back to the market. Due to the non-tradability of spot electricity, the electricity market is incomplete and the standard arbitrage-free pricing arguments of finance are inapplicable.

Article [III] assumes that an equivalent martingale measure $Q$ exists and market participants price derivatives instruments with $Q$. The measure $Q$ is not unique, as the non-tradability of electricity leads to an incomplete market. Nevertheless, theoretical prices for derivatives with normalised payoffs $\tilde{F}(\omega)$ are given by

$$f(t, \omega) = \mathrm{E}^Q[\tilde{F}(\omega)|\mathcal{F}_t].$$

The article does not try to explicitly quantify the derivative prices but focuses on available general results and theoretical price relations.

Article [III] shows that forward prices converge to the market expectations of spot prices at the end of their trading period. The result is well in line with the efficient market hypothesis in finance. The article proves that the expected spot prices, and hence forward prices, are different from the actual price realisations, $P$-almost surely. The article summarises these results to show that the pricing measure $Q$ includes the market expectations of future forward price movements.

Electricity forward contracts are financial securities. They can be traded despite the non-tradability of spot electricity. Article [III] concludes that the pricing of derivatives on electricity forward contracts corresponds to the pricing of derivatives on forwards in finance. In particular, the difficulties of pricing European call options on forward contracts are no different from financial markets.

Article [III] provides a few arbitrage-free price relations between spot electricity derivative prices. The article shows that put and call options on spot price averages, i.e. *Asian* options, follow the put-call parity found in other financial markets. The article presents the special characteristics of swing option pricing, and an arbitrage-free lower bound for the price. The lower bound is based on electricity forward prices. The article shows that, with similar parameters, the swing option with greater flexibility can have a worse payoff than an Asian option. The reason for this is that the terms of the swing option can force an exercise at unfavourable times. The article introduces several open problems for further study.

Actual market prices and price relations from the Nordic market support the theoretical results. European option prices for the forward contracts are higher in the market than the theoretical Black and Scholes prices. Historical data that provide model parameters does not predict the future but the market prices do include the expectations of market participants as to the future. Also, market frictions mean that market participants require higher profits for the risks involved in options. Finally, the article presents possible hedging alternatives for an electricity end user and the consequences of hedging strategies.

## 3.4 Risk management

Article [IV] considers a risk-averse market participant's risk management problem in competitive electricity markets. The participant holds a portfolio that consists of financial derivative instruments, physical assets, and other commitments dependent on electricity market prices. Electricity spot price characteristics and complex derivative instruments in the portfolio mean that no analytical solutions exist for the optimal portfolio selection problem. Article [IV] presents a quantitative method that captures the main characteristics of the problem. The method is computationally efficient and gives practical solutions to market participants' optimal portfolio selection problems.

Article [IV] formulates the risk management problem as an optimal portfolio selection problem. The objective of the optimal portfolio selection is to maximise the utility of wealth. The model is static in the sense that initial decisions cannot be adjusted. However, the model captures spot price dynamics and the cash-flows linked to them during the optimisation period. The initial portfolio is fixed $\pi_0 \in \mathbb{R}^{n+1}$. The optimal portfolio selection problem is to adjust $\pi_0$ by $\theta \in \mathbb{R}^{n+1}$ so that the resulting portfolio $\pi = \pi_0 + \theta$ maximises the expected utility at the end of a fixed time period. Derivatives and assets in the portfolio yield a payoff $F(\omega) \in \mathbb{R}^{n+1}$ during the optimisation period $[0, T]$. These payoffs accumulate the wealth $W(\pi, F(\omega)) := \pi^{\mathrm{T}} F(\omega)$ at the end of the time period. The wealth is reduced by the initial costs of derivatives and assets. The portfolio selection problem is

$$\max_{\theta \in \Theta} \mathrm{E}[U(W(\pi, F(\omega)))], \tag{3.1}$$

where the $\Theta \in \mathbb{R}^{n+1}$ gives the portfolio constraints. The initial decision is to choose $\theta$. Certain payoffs $F(\omega)$ realise and reduce or increase the wealth. The expected utility models the participants' risk preferences.

If the change in portfolio wealth is small, an approximation reduces the computational requirements. The approach taken in article [IV] corresponds with the derivation of the Arrow-Pratt measure of risk aversion. A Taylor's expansion approximates the utility function around the initial position, i.e. the wealth given by the initial portfolio $\pi_0$. The approximation of problem (3.1) is

$$\max_{\theta \in \Theta} \mathrm{E}[\theta^{\mathrm{T}} \frac{\partial U}{\partial \pi}(\pi_0^{\mathrm{T}} F(\omega)) + \frac{1}{2}\theta^{\mathrm{T}} \frac{\partial^2 U}{\partial \pi^2}(\pi_0^{\mathrm{T}} F(\omega))\theta], \tag{3.2}$$

with a ignorable constant term $\mathrm{E}[U(W(\pi_0, F(\omega)))]$ and an error term $\mathrm{E}[\mathcal{O}(\theta^3)]$. Monte Carlo simulation provides a numerical approximation of the expectation of the Taylor's expansion. The problem thus becomes a deterministic quadratic optimisation problem with given portfolio constraints $\theta \in \Theta$. If $\Theta$ is a convex set, then solving the deterministic quadratic optimisation problem is straightforward.

Solution of (3.2) requires the expectations of derivative and asset payoffs $F(\omega) \in \mathbb{R}^{n+1}$. Monte Carlo simulation over the optimisation period produces payoffs based on the simulated market prices and other relevant stochastic factors. These payoffs also give approximations of market prices of the derivatives and assets in the portfolio. Inter-temporal portfolio adjustments would require the computation of conditional asset prices in each state. Article [IV] is restricted to a static optimal portfolio selection framework due to the computational challenges involved in such an approach.

To carry out the Monte Carlo simulation, article [IV] presents an electricity spot price model. The expected spot price is equal to the prevailing forward prices,

$$\mathrm{E}[S(\tau,\omega)|\mathcal{F}_t] = f(t;\tau), \tag{3.3}$$

where $f(t;\tau)$ is the forward price for delivery time $\tau$ at time $t$. Remaining uncertainty around the expected value is log-normal[5]. A calibration of the model parameters makes the expected spot prices equal to the electricity forward prices. Model calibration prevents the optimisation of exploiting differences between market forward prices and simulated prices.

Monte Carlo simulation of a fixed portfolio over time gives an estimate of the distribution of portfolio wealth. Hence, a profit-at-risk[6] measure, or any other value from the wealth distribution, is directly available. Article [IV] presents numerical solutions for an electricity end user and generator in the Nordic market. The results of optimising with respect to a risk-averse utility function are consistent with the reduction of risk as shown by the profit-at-risk measure.

# 4 Discussion

The purpose of mathematical finance is to explain the phenomena observed in financial markets. Typical questions relate to market price modelling, derivative instrument pricing, and optimal portfolio selection. The same questions are relevant in competitive electricity markets.

Stock price modelling in finance has relied on convenient mathematical models. However, economic explanations of the price processes are important in applications. For example, optimal portfolio selection or derivative pricing in incomplete markets should not be based on arbitrary characteristics of price models. It is possible to construct models and explain electricity spot prices on the basis of market fundamentals. A similar problem exists in the modelling of forward price dynamics. The inclusion of economic and market fundamental factors in the forward price analysis should help to increase the explanatory power of forward price models. The variety of models in the literature calls for a comparative study to assess the performance of approaches in explaining observed price movements.

Results of mathematical finance are unavailing in derivative instrument pricing in competitive electricity markets. The main reason is the non-storability of electricity that implies market incompleteness. In incomplete

[5]The article erroneously states that the returns of the prices are log-normally distributed. This is incorrect: The assumption is that prices are log-normally distributed.

[6]The article uses the term value-at-risk in an identical meaning.

markets, it is impossible to construct arbitrage-free pricing rules that are the basis of derivative pricing in financial markets. Derivative prices based on some subjective price models have no theoretical relation to market prices. The possibilities of stating objective price relations in electricity markets are limited. Recent promising results in finance arise from the possibility of using other observed market prices to create arbitrage-free derivative price intervals. Within such an interval, prices can be selected on the basis of investors' preferences, market views, or other commitments.

The prevalent business principles of finance are similar in electricity markets. Some concepts introduced in financial markets are therefore directly usable in electricity markets. A careful application of optimal portfolio selection and risk management is possible following the approaches in financial markets. Likewise, risk measurement is as important, or even more important, than in financial markets, due to the large electricity spot price uncertainty. Dynamic optimal portfolio selection and development of risk measures in electricity markets are subject to further research.

The perspective of finance is that of an individual investor. The use of developed models and methods in business determines the usefulness of the mathematical explanations. Globalisation and the resulting increase in competition will heighten the need for effective applications of advanced theoretical models with a sound economic basis.

# References


Amundsen, E. S. and B. Singh. Developing futures markets for electricity in Europe. *Energy Journal*, 13(3):95–112 (1992).

Anderson, E. J. and A. B. Philpott. Optimal offer construction in electricity markets. *Mathematics of Operations Research*, 27(1):82–100 (2002).

Arrow, K. J. Aspects of the theory of risk-bearing. Lecture at Yrjö Hahnsson Foundation in Helsinki, Finland (1965).

Artzner, P.; F. Delbaen; J.-M. Eber; and D. Heath. Coherent measures of risk. *Mathematical Finance*, 9(3):203–228 (1999).

Bachelier, L. Théorie de la spéculation. *Annales Scientifiques de l'Ecole Normale Supérieure* (1900). (English Translation: In P. H. Cootner, editor, *Random Character of Stock Market Prices*, Massachusetts Institute of Technology, pages 17–78, 1964).

Barlow, M. A diffusion model for electricity prices. *Mathematical Finance*, 12(4):287–298 (2002).

Baumol, W. J. An expected gain-confidence limit criterion for portfolio selection. *Management Science*, 10(1):174–182 (1963).

Benth, F. E. On arbitrage-free pricing of weather derivatives based on fractional Brownian motion. *Applied Mathematical Finance*, 10(4):303–324 (2003).

Benth, F. E.; L. Ekeland; R. Hauge; and B. F. Nielsen. On arbitrage-free pricing of forward contracts in energy markets. *Applied Mathematical Finance*, 10(4):325–336 (2003).

Bessembinder, H. and M. L. Lemmon. Equilibrium pricing and optimal hedging in electricity forward. *Journal of Finance*, 57(3):1347–1382 (2002).

Bielecki, T. R. and S. R. Pliska. Risk-sensitive dynamic asset management. *Applied Mathematics & Optimization*, 39(3):337–360 (1999).

Bjorgan, R.; C.-C. Liu; and J. Lawarree. Financial risk management in a competitive electricity market. *IEEE Transactions on Power Systems*, 14(4):1285–1291 (1999).

Bjorgan, R.; H. Song; C.-C. Liu; and R. Dahlgren. Pricing flexible electricity contracts. *IEEE Transactions on Power Systems*, 15(2):477–482 (2000).

Björk, T. and C. Landén. On the construction of finite dimensional realizations for nonlinear forward rate models. *Finance and Stochastics*, 6(3):303–331 (2002).

Bjørkvoll, T.; S.-E. Fleten; M. P. Nowak; A. Tomasgard; and S. W. Wallace. Power generation planning and risk management in a liberalised market. In *2001 IEEE Porto Power Tech Proceedings*, pages 426–431 (2001).

Black, F. The pricing of commodity contracts. *Journal of Financial Economics*, 3(1):167–179 (1976).

Black, F. and M. Scholes. The pricing of options and corporate liabilities. *The Journal of Political Economy*, 81(3):637–654 (1973).

Botnen, O. J.; O. Frøystein; A. Haugstad; A. Johannesen; and S. Kroken. Coordination of hydropower and thermal generation systems for enhanced multinational resource utilization. In E. Broch and D. K. Lysne, editors, *Hydropower '92, Balkema, Rotterdam* (1992).

Boyle, P. Options: A Monte Carlo approach. *Journal of Financial Economics*, 4(3):323–338 (1977).

Boyle, P.; M. Broadie; and P. Glasserman. Monte Carlo methods for security pricing. *Journal of Economic Dynamics and Control*, 21(8/9):1267–1321 (1997).

Brennan, M. J. and E. S. Schwartz. Finite difference methods and jump processes arising in the pricing of contingent claims: A synthesis. *Journal of Financial and Quantitative Analysis*, 13(3):461–474 (1978).

Brennan, M. J. and E. S. Schwartz. Evaluating natural resource investments. *Journal of Business*, 58(2):135–157 (1985).

Burger, M.; B. Klar; A. Müller; and G. Schindlmayr. A spot market model for pricing derivatives in electricity markets. *Quantitative Finance*, 4(1):109–122 (2004).

Carr, P. and D. B. Madan. Option valuation using the fast Fourier transform. *Journal of Computational Finance*, 2(4):61–73 (1998).

Carr, P. and L. Wu. Time-changed Lévy processes and option pricing. *Journal of Financial Economics*, 17(1):113–141 (2004).

Courtadon, G. A more accurate finite difference approximation for the valuation of options. *Journal of Financial and Quantitative Analysis*, 17(5):697–705 (1982).

Cox, J. C.; J. E. Ingersoll; and S. A. Ross. A theory of the term structure of interest rates. *Econometrica*, 53(2):385–408 (1985).

Cox, J. C. and S. A. Ross. The valuation of options for alternative stochastic processes. *Journal of Financial Economics*, 3(1):145–166 (1976).

Cox, J. C.; S. A. Ross; and M. Rubinstein. Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3):229–263 (1979).

Cvitanic, J. Theory of portfolio optimization in markets with frictions. In E. Jouini; J. Cvitanic; and M. Musiela, editors, *Handbooks in Mathematical Finance: Option Pricing, Interest Rates and Risk Management.* Cambridge University Press (2001).

Davis, M. Pricing weather derivatives by marginal value. *Quantitative Finance*, 1(3):305–308 (2001).

Davison, M.; C. L. Anderson; B. Marcus; and K. Anderson. Development of a hybrid model for electrical power spot prices. *IEEE Transactions on Power Systems*, 17(2):257–264 (2002).

Deaton, A. and G. Laroque. Competitive storage and commodity price dynamics. *The Journal of Political Economy*, 104(5):896–923 (1996).

Delbaen, F. and W. Schachermayer. A general version of the fundamental theorem of asset pricing. *Mathematische Annalen*, 300(3):463–520 (1994).

Delbaen, F. and W. Schachermayer. The fundamental theorem of asset pricing for unbounded stochastic processes. *Mathematische Annalen*, 312(2):215–250 (1998).

Deng, S.-J. Pricing electricity derivatives under alternative spot price models. In *Proceedings of the 33rd Hawaii International Conference on System Sciences* (2000).

Deng, S.-J.; B. Johnson; and A. Sogomonian. Exotic electricity options and the valuation of electricity generation and transmission assets. *Decision Support Systems*, 30(3):383–392 (2001).

Dixit, A. and R. Pindyck. *Investment under Uncertainty.* Princeton University Press (1994).

Duffie, D. and S. Gray. Volatility in energy prices. In R. Jameson, editor, *Managing Energy Price Risk*, pages 39–55. Risk Publications (1995).

Duffie, D.; J. Pan; and K. Singleton. Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, 68(6):1343–1376 (2000).

Eydeland, A. and H. Geman. Fundamentals of electricity derivative pricing. In R. Jameson, editor, *Energy Modelling and Management of Uncertainty*, pages 35–43. Risk books (1999).

Fama, E. F. The behavior of stock prices. *Journal of Business*, 38(1):34–105 (1965).

Fama, E. F. Efficient capital markets: A review of theory and empirical work. *Journal of Finance*, 25(2):383–417 (1970).

Fleten, S.-E.; S. W. Wallace; and W. T. Ziemba. Portfolio management in a deregulated hydropower based electricity market. In E. Broch; D. Lysne; N. Flatbø; and E. Helland-Hansen, editors, *Hydropower '97*, pages 197–204. A.A. Balkema (1997).

Fleten, S.-E.; S. W. Wallace; and W. T. Ziemba. Hedging electricity portfolios via stochastic programming. In C. Greengard and A. Ruszczynski, editors, *Decision Making Under Uncertainty: Energy and Power, Vol. 128 of IMA Volumes on Mathematics and Applications*, pages 71–94. Springer-Verlag (2002).

Föllmer, H. and M. Schweizer. A microeconomic approach to diffusion models for stock prices. *Mathematical Finance*, 3(1):1–23 (1993).

Friedman, M. and L. J. Savage. The utility analysis of choices involving risk. *The Journal of Political Economy*, 56(4):279–304 (1948).

Föllmer, H. and A. Schied. *Stochastic Finance: An Introduction in Discrete Time*. Walter de Gruyter, Berlin (2002).

Gedra, T. W. Optional forward contracts for electric power markets. *IEEE Transactions on Power Systems*, 9(4):1766–1773 (1994).

Ghosh, K. and V. C. Ramesh. An options model for electric power markets. *Electrical Power & Energy Systems*, 19(2):75–85 (1997).

Gibson, R. and E. S. Schwartz. Stochastic convenience yield and the pricing of oil contingent claims. *Journal of Finance*, 45(3):959–976 (1990).

Hakansson, N. H. and W. T. Ziemba. Capital growth theory. In R. A. Jarrow; V. Maksimovic; and W. T. Ziemba, editors, *Handbooks in Operations Research and Management Science: Finance, Volume 9*, pages 65–86. North Holland, Amsterdam (1995).

Hanoch, G. and H. Levy. The efficiency analysis of choices involving risk. *The Review of Economic Studies*, 36(3):335–346 (1969).

Harrison, J. M. and D. Kreps. Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory*, 20(3):381–408 (1979).

Harrison, J. M. and S. R. Pliska. Martingale and stochastic integrals in the theory of continuous trading. *Stochastic Process and Their Applications*, 11(3):215–260 (1981).

Haugstad, A. and O. Rismark. Price forecasting in an open electricity market based on system simulation. In *EPSOM'98, Zurich, Switzerland* (1998).

Heath, D.; R. Jarrow; and A. Morton. Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuation. *Econometrica*, 60(1):77–105 (1992).

Hinz, J. An equilibrium model for electricity auctions. *Applicationes Mathematicae*, 30:243–249 (2003).

Hobbs, B. F.; C. B. Metzler; and J.-S. Pang. Strategic gaming analysis for electric power networks: An MPEC approach. *IEEE Transactions on Power Systems*, 15(2):638–645 (2000).

Huisman, R. and R. Mahieu. Regime jumps in electricity prices. *Energy Economics*, 25(5):425–434 (2003).

Hull, J. C. and A. White. Pricing interest rate derivative securities. *The Review of Financial Studies*, 3(1):573–592 (1990a).

Hull, J. C. and A. White. Valuing derivative securities using the explicit finite difference method. *Journal of Financial and Quantitative Analysis*, 25(1):87–100 (1990b).

Jacka, S. D. Optimal stopping and the American put. *Mathematical Finance*, 1(2):1–14 (1991).

Johnsen, T. A. Demand, generation and price in the Norwegian market for electric power. *Energy Economics*, 23(3):227–251 (2001).

Johnson, H. E. An analytic approximation for the American put price. *Journal of Financial and Quantitative Analysis*, 18(1):141–148 (1983).

Jorion, P. *Value at Risk: The New Benchmark for Controlling Market Risk*. McGraw-Hill (1997).

Kallberg, J. G. and W. T. Ziemba. Comparison of alternative utility functions in portfolio selection problems. *Management Science*, 29(11):1257–1276 (1983).

Kallio, M. and W. T. Ziemba. Arbitrage pricing simplified. In *Stochastic Programming E-Print Series* (2003). URL http://www.speps.info.

Karatzas, I. Optimization problems in the theory of continuous trading. *SIAM Journal of Control and Optimization*, 27(6):1221–1259 (1989).

Karatzas, I. and S. E. Shreve. *Brownian Motion and Stochastic Calculus*. Springer-Verlag, second edition (1988).

Karatzas, I. and S. E. Shreve. *Methods of Mathematical Finance*. Springer-Verlag (1998).

Kawai, M. Spot and futures prices of nonstorable commodities under rational expectations. *Quarterly Journal of Economics*, 98(2):235–254 (1983).

Kaye, R. J.; H. R. Outhred; and C. H. Bannister. Forward contracts for the operation of an electricity industry under spot pricing. *IEEE Transactions on Power Systems*, 5(1):46–52 (1990).

Keppo, J. and M. Räsänen. Pricing of electricity tariffs in competitive markets. *Energy Economics*, 21(3):213–223 (1999).

King, A. J. Duality and martingales: A stochastic programming perspective on contingent claims. *Mathematical Programming, Series B*, 91(3):543–562 (2002).

Koekebakker, S. and F. Ollmar. Forward curve dynamics in the Nordic electricity market (2001). URL http://www.nhh.no/for/dp/2001/2101.pdf, unpublished.

Lari-Lavassani, A.; M. Simchi; and A. Ware. A discrete valuation of swing options. *Canadian Applied Mathematics Quarterly*, 9(1):35–74 (2001).

Lee, R. Option pricing by transform methods: Extensions, unification, and error control. *Journal of Computational Finance*, 7(3) (2004).

Leland, H. E. Agency costs, risk management, and capital structure. *Journal of Finance*, 53(4):1213–1243 (1998).

Lemming, J. Price modelling for profit at risk management. In D. W. Bunn, editor, *Modelling Prices in Competitive Electricity Markets*, pages 287–306. Wiley (2004).

Litterman, R. and J. Scheinkman. Common factors affecting bond returns. *Journal of Fixed Income*, 1(1):54–61 (1991).

Lucas, R. E. Expectations and the neutrality of money. *Journal of Economic Theory*, 4:103–124 (1972).

Lucas, R. E. Econometric policy evaluation: A critique. *Carnegie-Rochester Conference Series on Public Policy*, 1:19–46 (1976).

Lucia, J. J. and E. S. Schwartz. Electricity prices and power derivatives: Evidence from the Nordic power exchange. *Review of Derivatives Research*, 5(1):5–50 (2002).

Luenberger, D. G. A preference foundation for log mean-variance criteria in portfolio choice problems. *Journal of Economic Dynamics and Control*, 17(5/6):887–906 (1993).

MacMillan, L. W. An analytical approximation for the American put prices. *Advances in Futures and Options Research*, 1(3):119–139 (1986).

Mandelbrot, B. Forecasts of future prices, unbiased markets and martingale models. *Journal of Business*, 39(1):242–255 (1966).

Markowitz, H. M. Portfolio selection. *The Journal of Finance*, 7(1):77–91 (1952a).

Markowitz, H. M. The utility of wealth. *The Journal of Political Economy*, 60(2):151–158 (1952b).

Markowitz, H. M. *Portfolio selection: Efficient diversification of investments*. John Wiley & Sons Ltd. (1959).

Merton, R. C. Lifetime portfolio selection under uncertainty: The continuous time case. *Review of Economics and Statistics*, 51(3):247–257 (1969).

Merton, R. C. Optimum consumption and portfolio rules in a continuous time model. *Journal of Economic Theory*, 3:373–413 (1971).

Merton, R. C. Theory of rational option pricing. *Bell Journal of Economics and Management Science*, 4(1):141–183 (1973).

Merton, R. C. Asymptotic theory of growth under uncertainty. *The Review of Economic Studies*, 42(3):375–393 (1975).

Merton, R. C. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics*, 3(1):125–144 (1976).

Miltersen, K. R. and E. S. Schwartz. Pricing options on commodity futures with stochastic term structures of convenience yields and interest rates. *Journal of Financial and Quantitative Analysis*, 33(1):33–59 (1998).

Mo, B. and A. Gjelsvik. Simultaneous optimisation of withdrawal from a flexible contract and financial hedging. In *14th Power Systems Computation Conference*. Sevilla, Spain (2002). URL http://www.pscc02.org/.

Mo, B.; A. Gjelsvik; and A. Grundt. Integrated risk management of hydro power scheduling and contract management. *IEEE Transactions on Power Systems*, 16(2):216–221 (2001).

Muth, J. F. Rational expectations and the theory of price movements. *Econometrica*, 29(3):315–335 (1961).

Pilipovic, D. *Energy Risk*. Irwin Professional Publishing (1997).

Pratt, J. W. Risk aversion in the small and in the large. *Econometrica*, 32(1/2):122–136 (1964).

Rabin, M. Risk aversion and expected-utility theory: A calibration theorem. *Econometrica*, 68(5):1281–1292 (2000).

Rabin, M. and R. H. Thaler. Anomalies: Risk aversion. *The Journal of Economic Perspectives*, 15(1):219–232 (2001).

Ross, S. A. The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, 3:343–362 (1976).

Samuelson, P. A. Proof that properly anticipated prices fluctuate randomly. *Industrial Management Review*, 6(2):41–50 (1965a).

Samuelson, P. A. Rational theory of warrant pricing. *Industrial Management Review*, 6(2):13–31 (1965b).

Samuelson, P. A. Lifetime portfolio selection by dynamic stochastic programming. *Review of Economics and Statistics*, 51(3):239–246 (1969).

Schwartz, E. S. The stochastic behavior of commodity prices: Implications for valuation and hedging. *Journal of Finance*, 52(3):923–973 (1997).

Simonsen, I. Measuring anti-correlations in the Nordic electricity spot market by wavelets. *Physica A*, 283(3):597–606 (2003).

Skantze, P. L. and M. D. Ilic. *Valuation, hedging and speculaton in competitive electricity markets: A fundamental approach*. Kluwer Academic Publishers (2001).

Smith, C. W. and R. M. Stulz. The determinants of firms' hedging policies. *Journal of Financial and Quantitative Analysis*, 20(2):391–405 (1985).

Smith, J. and K. McCardle. Valuing oil properties: Integrating option pricing and decision analysis approaches. *Operations Research*, 46(2):198–217 (1998).

Steinbach, M. C. Markowitz revisited: Mean-variance models in financial portfolio analysis. *SIAM Review*, 43(1):31–85 (2001).

Sundaresan, S. M. Continuous-time methods in finance: A review and an assessment. *Journal of Finance*, 55(4):1569–1622 (2000).

Thompson, A. C. Valuation of path-dependent contingent claims with multiple exercise decisions over time: The case of take-or-pay. *Quantitative Analysis*, 30(2):271–293 (1995).

Tobin, J. Estimation of relationships for limited dependent variables. *Econometrica*, 26(1):24–36 (1958).

Tomek, W. G. and H. H. Peterson. Risk management in agricultural markets: A review. *The Journal of Futures Markets*, 21(10):953–985 (2001).

Tseng, C.-L. and G. Barz. Short-term generation asset valuation - a real options approach. *Operations Research*, 50(2):297–310 (2002).

Vasicek, O. A. An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5(2):177–188 (1977).

Visudhiphan, P. and M. D. Ilic. Dynamic games-based modeling of electricity markets. In *Proceedings of the IEEE PES Winter Meeting*, pages 275–281 (1999).

von Neumann, J. and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press (1944).

Weron, R. Energy price risk management. *Physica A*, 285:127–134 (2000).

Weron, R.; M. Bierbrauer; and S. Trück. Modeling electricity prices: Jump diffusion and regime switching (2004a). Forthcoming in *Physica A*.

Weron, R. and B. Przybyłowicz. Hurst analysis of electricity price dynamics. *Physica A*, 283:462–468 (2000).

Weron, R.; I. Simonsen; and P. Wilman. Modeling highly volatile and seasonal markets: evidence from the Nord Pool electricity market. In H. Takayasu, editor, *The Application of Econophysics*, pages 182–191. Springer-Tokyo (2004b).

Williams, J. C. and B. D. Wright. *Storage and Commodity Markets*. Cambridge University Press (1991).

Working, H. The theory of the price of storage. *The American Economic Review*, 39(6):1254–1262 (1949).

(continued from the back cover)

HELSINKI UNIVERSITY OF TECHNOLOGY INSTITUTE OF MATHEMATICS
RESEARCH REPORTS

The list of reports is continued inside. Electronical versions of the reports are available at *http://www.math.hut.fi/reports/* .

A477 Tuomo T. Kuusi
Moser's Method for a Nonlinear Parabolic Equation
October 2004

A476 Dario Gasbarra , Esko Valkeila , Lioudmila Vostrikova
Enlargement of filtration and additional information in pricing models: a Bayesian approach
October 2004

A473 Carlo Lovadina , Rolf Stenberg
Energy norm a posteriori error estimates for mixed finite element methods
October 2004

A472 Carlo Lovadina , Rolf Stenberg
A posteriori error analysis of the linked interpolation technique for plate bending problems
September 2004

A471 Nuutti Hyvönen
Diffusive tomography methods: Special boundary conditions and characterization of inclusions
April 2004

ISBN 951-22-7323-3
ISSN 0784-3143