MATHEMATICS COURSE Y3, EXERCISE 1

16.9-20.9

Beginning of the week

1. Let $P_n(x)$ be n degree polynomial (Taylor expansion) for function $f(x)$ at the point x_0. Determine $P_n(x)$ and the upper bounded function for the error at the interval $[a, b]$, i.e. find $g(x)$ such that $|R_n(x)| \le g(x), \ x \in [a, b]$. Here $f(x) = \sin x, \ x_0 = \pi/4, \ n = 6, \ x \in [0, \pi/2]$.

2. Calculate the summations of the following series.
$a) \ \sum_{n=3}^{\infty} \frac{1}{(2n-3)(2n-1)} \quad b) \sum_{n=0}^{\infty} e^{-n} \quad c) \sum_{n=1}^{\infty} (-1)^n (3/4)^n.$

3. Classify the following series as absolutely convergent, conditional convergent, or divergent.
$a) \ \sum_{n=1}^{\infty} (-1)^n \frac{n!\ n!}{(2n)!} \ b) \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n(\ln n)^2} \ c) \sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}.$
(Hint: Use suitable tests/theorem like ratio test, root test, integral test, majorant/minorant test or Leibniz.)

4. a) Show that the subharmonic series $\sum_{n=1}^{\infty} \frac{1}{n^p}, \ 0 < p < 1$ divergents by using comparison test method. (comparing it with the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$).

b) Give two examples to show that the convergent/divergent conclusion cannot be obtained by using the root test method on the positive term series $\sum_{n=1}^{\infty} a_n, \ a_n \ge 0$ if $\lim_{n\to\infty}(a_n)^{1/n} = 1$.

5. Determine the radii and the intervals for the power series $\sum_{n=1}^{\infty} \frac{(x+4)^n}{n3^n}$, where the series 1) convergents uniformly, 2) convergents conditionally, 3) divergents.

6. Calculate e^{-2} by using Maclaurin polynomial in such way that the remainder (error) $|error| < 10^{-5}$. Use the Leibnitz theorem to estimate the error.

End of the week

Before starting the exercises let's look at some basic MAPLE instructions.

1. To use Taylor polynomial for the function $f(x)$ at the point x_0, we put the command $sarja := taylor(f(x), x0, n)$. You can get its Taylor polynomial (without the remainder) by $convert(sarja, polynom)$;

To make the Maclaurin polynomial for a function, show the graphs of the function, the polynomials in the same picture, you can give the command
$taylor, convert(\dots, polynom), plot.$
or
$t0 := taylor(exp(x), x = 0, 1); t1 := taylor(exp(x), x = 0, 2); \dots,$
$plot(exp(x), t1, t2, \dots, x = a, \dots, b)$. (give suitable values for a and b.)

Try the following

$x :=' x'; N := 5; jono := seq(taylor(exp(x),\ \ x = 0, n + 1),\ \ n = 0..N);$

(Note: You can drop out the remainder by using $convert(\dots, polynom)$. Then you can draw the picture)

$plot(\{exp(x), jono\},\ \ x = -6..2,\ \ y = -2..8);$

2. • Apply the above instructions to the problem 1 (Exercise 1, beginning of the week). Draw the graphs of the upper bounded function $g(x)$ and the remainder $f(x) - P_n(x)$ on the same picture.

• Do the above exercise for the function $f(x) = \ln \cos x,\ x0 = 0,\ n = 3, x \in [0, \pi/6]$. Note that $diff(f, x\$n)$ calculates the nth derivative for the function $f(x)$.

3. Compute the partial summation $\sum_{k=1}^{\infty} 1/k^3$, when $n = 1, \dots, 10$;

Hint:
$sum(1/k^3,\ \ k = 1..1);$
$sum(1/k^3,\ \ k = 1..2);$
$\dots$

You'll notice that $evalf$ is needed as you get tired you can try
$for\ \ n\ \ to\ 10\ do\ evalf(sum(1/k^3,\ \ k = 1..n))\ \ od;$

Compute the partial sums for $s_{10}, s_{20}, s_{40}, s_{80}$. Study its convergent behavior (assume that the same trend continues).

The sum of the series can be obtained by using the sum command.

Hint: You can try $s_{10} := \ldots,\ s_{20} := \ldots, \ldots$ first. At the end try the command

$s := Sum(1/k^3, k = 1..n);$
? *seq*
$jono := evalf(seq(subs(n = kk, s), kk = [10, 20, 40, 80]));$

4. If it's possible you can make the following Maple exercises.

1. Maple exercises

Y3, Autumn, 1996. The following is the collection of Maple exercises which are aimed to practise your Maple skill. If you have some problems you can make inquiries on the exercise classes and if it is needed some solutions will be posted on the homepage of this course. We just got the Maple license but the installation can be delayed a bit. Anyway, Maple was already installed into the computer room U352 and Marrintalo. As the lecture starts Maple will be installed into HP and $\alpha-$(alpha) workstations. Maple can be installed into all Windows, Macintosh, linux etc. Students can buy it from the computer center by a special price. The price will be announced later.

At this moment the new Maple version on Windows and Mac is V.4. V.3 is very good too. The big difference between V.4 and V.3 is the user interface.

To invoke Maple on the workstations, you can use the commands:

use maple
xmaple or
maple (for the text terminal)

On the text terminal the graphical user interface shows the formula and pictures more beautifully.

Some basic Maple commands

$>?$
$>?library$
$>?index$
$plot, subs, eval, evalf, value, diff, int, solve$ (you can make an inquiry for all these

functions by ? command)
$> x := arvo$;(assign the value)
$> a * x + b = c$;(define the relation)

Documents

You can find some related documents on the web site
http://www.math.hut.fi/teaching/y3/opemate/. like
Holms: An introduction Guide to Maple (15 p.). Also the copy of it is at the room U352. Other recommended literatures are: 1) Maple V Learning Guide, 2) Maple V Programming Guide, 3) Maple V Handbook. You can read Waterloo-Maple on
http://daisy.waterloo.ca/
and Holms on
http://www.math.hut.fi/teaching/numsym/intro.

In the following, I 'll give some problems and everyone can try to solve them.

Problems

1. Visit the web site **http://www.math.hut.fi/teaching/y3/opemate/** for **Holms: An introduction Guide to Maple** and do the exercise on page 2.

2. Find the roots, minimum and maximum of the polynomial $p(x) = x^3 - 4x^2 + 4x - 1$. Do the following tests:

$p := x \wedge 3 - \ldots;$
$dp := diff(p, x);$
$?solve$
$?subs$

3. Compute the sums of a) $\sum_{k=1}^{1000} k$, b) $\sum_{k=1}^{\infty} 1/k^2$, c) $\sum_{k=1}^{n} k$, d) $\sum_{k=1}^{n} 1/k^2$.

$?sum$

4. Look at the function $h(x) = 1 - x + \sin x$. Treat it as a) expression b) function. Explain the meanings of $h(-2)$ on both situations and draw the graphs of h-function and its derivative on the interval [-5,5].

$expression$	$function$
$hlaus := 1 - x - \sin(x)$	$h := x- > ...$

Please read the Holms p13 if it is needed. You have to know the differences between the Maple-expression and the Maple-function. See the following formula in Mathemaica:

$hclaus = 1 - x - Sin[x]$
$h[x\_] := 1 - x + Sin[x]$

5. Try the following:

$Int(cos(x), x);\ \ value('');$
$Int(cos(x),\ \ x = a..b);\ \ value('')$(This is recommended. It can help to find the $int(cos(x),\ \ x = a..b)$ writing errors)

Further,

$int(sin(cos(x)), x)$(If Maple cannot solve it)
$int(sin(x)/x, x)$

6. Solve the initial value problem

$$y"(x) = 2x^2 \sin x^2 - \cos x^2,\ \ y(\sqrt{\pi}) = 2\sqrt{\pi},\ \ y'(\sqrt{\pi}) = 2$$

The solution can be obtained by integrating the right hand side.

$$y'(x) = 2 + \int_{\sqrt{\pi}}^{x} y"(t)dt$$

$$y(x) = 2\sqrt{\pi} + \int_{\sqrt{\pi}}^{x} y'(t)dt$$

Apply this idea to Maple we have:

$2 + Int(2 * t \wedge 2 * sin(t \wedge 2) - cos(t \wedge 2), t = sqrt(Pi)..x);$
$dy := value('');$(dy:=2+Int(...))
$dy := subs(x = t, dy);$(in dy, replace x by t)

You can continue to the end. Notice that you need **subs** and **eval** commands since Maple doesn't always evaluate the functions automatically.

7. Draw the graphs of the function

$$f(x) = \frac{x^3 + 1}{x^2 + x + 1}$$

and its 1th, 2nd derivatives into the same picture. How can you distinguish them? You can do as follows:

$?plot$
$?color$
$p1 := plot(f, x = -infinity..infinity, color = red)$:(use ':'. ';' leads to the problem)
$p2 := plot(f, x = -infinity..infinity, color = green)$:
$p3 := plot(f, x = -infinity..infinity, color = blue)$:
$with(plots)$:(drawing)
$display(\{p1, p2, p3\})$;(combining)

8. The volume of the balloon increases by $10cm^3/s$. How fast does its radius increase (cm^2/s) when its surface area is $200cm^2$.

Hint: $V(t) = 4/3\pi r(t)^3$, $A(t) = 4\pi r(t)^2$.
Derivatives are $V'(t) = ...$ Then $r'(t)$ can be expressed by $V'(t)$ and $r(t)$... Look at the following

$V := (4/3) * Pi * r(t) \wedge 3;\ A = 4 * Pi * r(t) \wedge 2;$
$yht1 := 10 = diff(V, t); yht2 := 200 = A;$