```
>
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Logaritmin derivaatta Cauchy-Riemannin avulla

```
> F:=evalc(log(x+I*y));
```

$$F := \frac{1}{2}\ln(x^2+y^2) + I\arctan(y, x)$$

```
> u:=Re(F);v:=Im(F);
```

$$u := \frac{1}{2}\ln(x^2+y^2)$$

$$v := \arctan(y, x)$$

C-R-yhtalot:

```
> simplify(diff(u,x)=diff(v,y));simplify(diff(v,x)=-diff(u,y));
```

$$\frac{x}{x^2+y^2} = \frac{x}{x^2+y^2}$$

$$-\frac{y}{x^2+y^2} = -\frac{y}{x^2+y^2}$$

... ovat siis voimassa.
Lasketaan derivaatta:

```
> DF:=simplify(diff(u,x)+I*diff(v,x));
```

$$DF := -\frac{-x+I\,y}{x^2+y^2}$$

Siis

```
> 'diff(log(z),z)'=conjugate(z)/(z*conjugate(z));
```

$$\frac{\partial}{\partial z}\log(z) = \frac{1}{z}$$

kuten asiaan kuuluu.

Harmoniset funktiot

```
> Delta:=(f,x,y)->diff(f,x,x)+diff(f,y,y);
```

$$\Delta := (f, x, y) \to \mathrm{diff}(f, x, x) + \mathrm{diff}(f, y, y)$$

```
> u:=3*x*y^2-x^3;'Delta(u,x,y)'=Delta(u,x,y);
```

$$u := 3\,x\,y^2 - x^3$$

$$\Delta(u, x, y) = 0$$

```
> vx:=-diff(u,y);   # 2. C-R-yht.
```

$$vx := -6\,x\,y$$

```
> v:=Int(vx,x)+A(y); # Int. vakio saa riippua y:sta
```

$$v := \int -6\,x\,y\,dx + \mathrm{A}(y)$$

```
> v:=value(v);
```

$$v := -3\,x^2\,y + \mathrm{A}(y)$$

```
> vy:=diff(v,y);
```

$$vy := -3\,x^2 + \left(\frac{\partial}{\partial y}\,\mathrm{A}(y)\right)$$

```
> simplify(vy=diff(u,x));  # C-R 1.
```

$$-3\,x^2 + \left(\frac{\partial}{\partial y}\,\mathrm{A}(y)\right) = 3\,y^2 - 3\,x^2$$

```
> dsolve(",A(y)); # Maple ei hoksaa x-kumoutumista mutta se ei
  haittaa dsolvea
```

$$\mathrm{A}(y) = y^3 + \_C1$$

```
> v:=subs(",v);
```

$$v := -3\,x^2\,y + y^3 + \_C1$$

```
> f:=u+I*v;
```

$$f := 3\,x\,y^2 - x^3 + I\,(-3\,x^2\,y + y^3 + \_C1)$$

```
> subs(x=(1/2)*(z+conjugate(z)),f);
```

$$3\left(\frac{1}{2}\,z + \frac{1}{2}\,\bar{z}\right)y^2 - \left(\frac{1}{2}\,z + \frac{1}{2}\,\bar{z}\right)^3 + I\left(-3\left(\frac{1}{2}\,z + \frac{1}{2}\,\bar{z}\right)^2 y + y^3 + \_C1\right)$$

```
> subs(y=(1/(2*I))*(z-conjugate(z)),");
```

$$-\frac{3}{4}\left(\frac{1}{2}\,z + \frac{1}{2}\,\bar{z}\right)(z - \bar{z})^2 - \left(\frac{1}{2}\,z + \frac{1}{2}\,\bar{z}\right)^3 + I\left(\frac{3}{2}\,I\left(\frac{1}{2}\,z + \frac{1}{2}\,\bar{z}\right)^2 (z - \bar{z}) + \frac{1}{8}\,I\,(z - \bar{z})^3 + \_C1\right)$$

```
> evalc(");
```

$$-z^3 + I\,\_C1$$

Hyvin Maple hallitsi taman sievennyksen, joka olisi kasin ollut hieman vaivalloinen.
(Kokeissa ei yleensä vaadita saattamista
z:n funktioksi.)