```
>
```

Harj. 12, LV, teht. 1

a) Crank-Nicolson

```
> n:=5:p:=n-1:u:='u':kmax:=5:dx:=0.2:
```

Alkuarvot:

```
> for j from 0 to n do u[j,0]:=evalf(sin(j*dx*Pi)) od:
```

Reuna-arvot:

```
> for k from 0 to kmax do u[0,k]:=0;u[5,k]:=0 od:
```

Nurkkia ei tarvitse keskiarvottaa, koska RE- ja AE- arvot ovat =0
Sitten vaan Crank-Nicolson.
Kullakin k:n arvolla saadaan p yhtaloa, joissa tuntemattomat u[j,k+1], j=1..p
(Kussakin yhtalossa on 3 tuntematonta: u[j-1,k+1],u[j,k+1],u[j+1,k+1] .)
Kutakin k-arvoa vastaavat p yhtaloa ratkaistaan samantien ja sitten muodostetaan
arvoa k+1 vastaavat p yhtaloa, ratkaistaan ne ja jatketaan:

```
> for k from 0 to kmax do
>   for j from 1 to p do
  yht[j,k]:=4*u[j,k+1]-u[j+1,k+1]-u[j-1,k+1]=u[j+1,k]+u[j-1,k]
  od:assign(solve({seq(yht[j,k],j=1..p)},{seq(u[j,k+1],j=1..p)}))od:
```

Kerataan saadut ratkaisut matriisiksi:

```
> with(linalg):
Warning, new definition for norm
Warning, new definition for trace
> um:=matrix(kmax+1,n+1,[seq(seq(u[j,k],j=0..n),k=0..kmax)] );
```

$$um := \begin{bmatrix} 0 & .5877852524 & .9510565165 & .9510565163 & .5877852522 & 0 \\ 0 & .3992737562 & .6460385084 & .6460385084 & .3992737562 & 0 \\ 0 & .2712207082 & .4388443244 & .4388443244 & .2712207082 & 0 \\ 0 & .1842361823 & .2981004050 & .2981004050 & .1842361823 & 0 \\ 0 & .1251488911 & .2024951595 & .2024951595 & .1251488911 & 0 \\ 0 & .08501177537 & .1375519420 & .1375519420 & .08501177537 & 0 \end{bmatrix}$$

```
>
```

b) Differenssimenet.

```
> n:=5:p:=n-1:u:='u':kmax:=4*5:dx:=0.2:
> r:=0.25:  # Otetaan hyva tapaus ensin.
> dt:=r*dx^2;
```

$$dt := .0100$$

Alkuarvot:

```
> for j from 0 to n do u[j,0]:=evalf(sin(j*dx*Pi)) od:
```

Reuna-arvot:

```
> for k from 0 to kmax do u[0,k]:=0;u[5,k]:=0 od:
```

```
> for k from 0 to kmax do for j from 1 to p do
  u[j,k+1]:=r*u[j-1,k]+(1-2*r)*u[j,k]+r*u[j+1,k] od od;
> u[1,3];
```

$$.4349673300$$

```
> with(linalg):
> umatr:=matrix(kmax+1,n+1,[seq(seq(u[j,k],j=0..n),k=0..kmax)]);
```

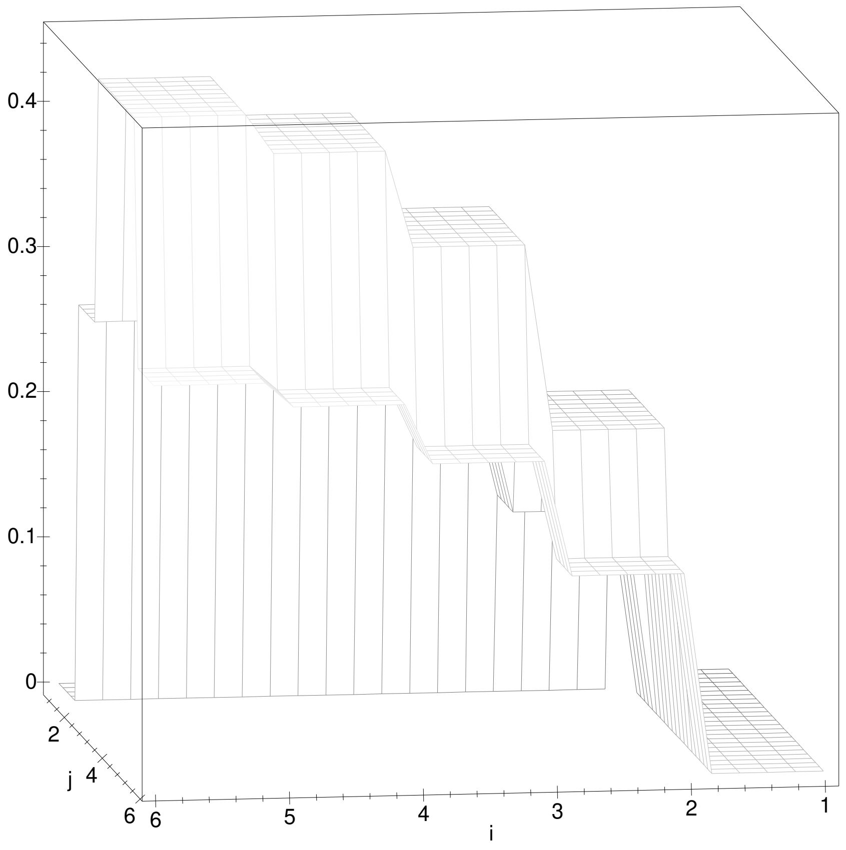
$$umatr := \begin{bmatrix} 0 & .5877852524 & .9510565165 & .9510565163 & .5877852522 & 0 \\ 0 & .5316567553 & .8602387005 & .8602387004 & .5316567552 & 0 \\ 0 & .4808880528 & .7780932142 & .7780932141 & .4808880527 & 0 \\ 0 & .4349673300 & .7037919238 & .7037919239 & .4349673299 & 0 \\ 0 & .3934316460 & .6365857754 & .6365857755 & .3934316460 & 0 \\ 0 & .3558622669 & .5757972431 & .5757972432 & .3558622669 & 0 \\ 0 & .3218804443 & .5208134991 & .5208134991 & .3218804443 & 0 \\ 0 & .2911435970 & .4710802355 & .4710802355 & .2911435970 & 0 \\ 0 & .2633418574 & .4260960760 & .4260960760 & .2633418574 & 0 \\ 0 & .2381949477 & .3854075214 & .3854075214 & .2381949477 & 0 \\ 0 & .2154493543 & .3486043780 & .3486043780 & .2154493543 & 0 \\ 0 & .1948757717 & .3153156221 & .3153156221 & .1948757717 & 0 \\ 0 & .1762667914 & .2852056595 & .2852056595 & .1762667914 & 0 \\ 0 & .1594348106 & .2579709426 & .2579709426 & .1594348106 & 0 \\ 0 & .1442101410 & .2333369097 & .2333369097 & .1442101410 & 0 \\ 0 & .1304392979 & .2110552176 & .2110552176 & .1304392979 & 0 \\ 0 & .1179834534 & .1909012377 & .1909012377 & .1179834534 & 0 \\ 0 & .1067170361 & .1726717916 & .1726717917 & .1067170361 & 0 \\ 0 & .09652646595 & .1561831027 & .1561831028 & .09652646598 & 0 \\ 0 & .08730900866 & .1412689435 & .1412689436 & .08730900869 & 0 \\ 0 & .07897174021 & .1277789598 & .1277789599 & .07897174025 & 0 \end{bmatrix}$$

Aika-askel on 1/4 CR-aika-askeleesta (CR:ssa r=1, tassa r=1/4). Jotta arvoja voidaan verrata, poimitaan joka 4. rivi:

```
> matr025:=submatrix(umatr,[1,5,9,13,17,21],1..6);
```

$$matr025 := \begin{bmatrix} 0 & .5877852524 & .9510565165 & .9510565163 & .5877852522 & 0 \\ 0 & .3934316460 & .6365857754 & .6365857755 & .3934316460 & 0 \\ 0 & .2633418574 & .4260960760 & .4260960760 & .2633418574 & 0 \\ 0 & .1762667914 & .2852056595 & .2852056595 & .1762667914 & 0 \\ 0 & .1179834534 & .1909012377 & .1909012377 & .1179834534 & 0 \\ 0 & .07897174021 & .1277789598 & .1277789599 & .07897174025 & 0 \end{bmatrix}$$

```
> plot3d(umatr[i,j]-matr025[i,j],i=1..6,j=1..6,axes=boxed);
```




```
> i:='i':j:='j':
```

Huono tapaus r=1

```
> n:=5:p:=n-1:u:='u':kmax:=5:dx:=0.2:
> r:=1:  # Otetaan nyt paha tapaus.
> dt:=r*dx^2;
```

$$dt := .04$$

Alkuarvot:

```
> for j from 0 to n do u[j,0]:=evalf(sin(j*dx*Pi)) od:
```

Reuna-arvot:

```
> for k from 0 to kmax do u[0,k]:=0;u[5,k]:=0 od:
```

```
> for k from 0 to kmax do for j from 1 to p do
  u[j,k+1]:=r*u[j-1,k]+(1-2*r)*u[j,k]+r*u[j+1,k] od od;
> u[1,3];
```

$$.1387572762$$

```
> with(linalg):
> umatr:=matrix(kmax+1,n+1,[seq(seq(u[j,k],j=0..n),k=0..kmax)]);
```

$$umatr := \begin{bmatrix} 0 & .5877852524 & .9510565165 & .9510565163 & .5877852522 & 0 \\ 0 & .3632712641 & .5877852522 & .5877852524 & .3632712641 & 0 \\ 0 & .2245139881 & .3632712643 & .3632712639 & .2245139883 & 0 \\ 0 & .1387572762 & .2245139877 & .2245139887 & .1387572756 & 0 \\ 0 & .0857567115 & .1387572772 & .1387572746 & .0857567131 & 0 \\ 0 & .0530005657 & .0857567089 & .0857567157 & .0530005615 & 0 \end{bmatrix}$$

Virhetta on selvasti, muttei tolkuttomasti ainakaan talla aikavalilla.