Helsinki University of Technology
Institute of Mathematics
Mat-1.3607 Stochastic Optimization, Fall 2006

Exercise 5
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Pennanen/Perkkiö

1 Verify that

$$E^P \varphi(\xi) = E^U \varphi(G(\omega)),$$

as claimed in Section 2.5.

2 Consider a conditionally Gaussian process and assume that the decision stages k coincide with the time steps t of the time series model. Write down the mapping g_k and give conditions under which they satisfy the conditions of Proposition 2.9.

3 Consider Example 2.1 in the case where there are two assets, a stock s^1 and a bank account s^2 as described in Example 2.8. Assume that the 2-dimensional process (η^1, η^2) is conditionally Gaussian. Let $\xi = (s^1, s^2)$ and assume that the decision stages k coincide with the time steps t of the time series model.

(a) Write down the mapping $g_k : (0,1)^{d_k} \mapsto \Xi_k$.

(b) Show that if the volatility matrices σ_t are invertible then g_k satisfy the conditions of Proposition 2.9.

4 Let ϕ be a density function on $\mathbb{R}$, that is, a nonnegative Lebesgue integrable function with $\int \phi = 1$, and let

$$\Phi(\varepsilon) = \int_{-\infty}^{\varepsilon} \phi(\epsilon) d\epsilon.$$

(a) Show that the measure $P = U\Phi$ has density ϕ.

(b) Given a procedure that generates random numbers uniformly distributed in $(0,1)$, how can you generate random numbers having distribution P?

5 How could you generalize the procedure in Problem 3 into a multivariate case? (Hint: Consider a recursion where you apply the univariate procedure of Problem 3 to one component at a time, first to the marginal distribution of the first component and then to the marginal distribution of the second component conditioned on the value of the first component, and so on.)