Exercise 1
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1 Prove Thm 1.1

2 Prove Thm 1.4

3 Verify the claims in Example 1.5

4 Let f be a lsc function and let K be a compact set in a topological space. Show that the problem

$$\text{minimize} \quad f \quad \text{over} \quad K$$

has a solution.

5 Let f be a τ_p-lsc convex function on L^p and let $K \subset L^p$ be τ_p-closed, convex and bounded. Show that if $p > 1$ then then the problem

$$\text{minimize} \quad f \quad \text{over} \quad K$$

has a solution.

6 Let $\mathcal{L}^0$ be the vector space of random variables with the vector space operations

$$(\alpha x)(\xi) = \begin{cases} 0 & \text{if } \alpha = 0, \\ \alpha x(\xi) & \text{otherwise,} \end{cases}$$

$$(x_1 + x_2)(\xi) = \begin{cases} 0 & \text{if } x_1(\xi) = -x_2(\xi) = \pm\infty, \\ x_1(\xi) + x_2(\xi) & \text{otherwise.} \end{cases}$$

Then the functions $x \mapsto Ex$ and $x \mapsto \operatorname{ess\,sup} x$ are convex on $\mathcal{L}^0$.