Helsinki University of Technology
Mathematics
Somersalo/Bingham

Mat-1.198 Scattering Theory

9th set of exercises, 9.4.2003

The exam will be on Tuesday, the 20th of May from 10.00 to 13.00 in U322.

1. Consider the one-dimensional potential scattering of the previous week. Derive the PML equation in the one-dimensional case: The stretching function

$$F : \mathbb{R} \to \mathbb{C}, \quad x \mapsto \begin{cases} x - i\tau(a-x), & x < a \\ x, & a \le |x| \le b \\ x + i\tau(x-b), & x > b \end{cases}$$

   maps the real axis to a curve in the complex plane. Here $(a,b) \supset [-M,M]$ is the truncated computation domain.

2. Consider the PML equation in radial coordinates, i.e., the computational region B is a disc,

$$\overline{D} \subset B = \{x \in \mathbb{R}^2 \mid |x| < R\} \subset \mathbb{R}^2.$$

   Given a stretching function $\tau : [0,\infty) \to [0,\infty)$, calculate the PML equation using the polar coordinate representation for the Laplacian.

3. The previous example allows us to write the PML equation as

$$\nabla \cdot A \nabla u + a k^2 u = 0,$$

   where $A = A(x) \in \mathbb{C}^{2\times 2}$, $a = a(x) \in \mathbb{C}$. Find this representation. In particular, what is A in Cartesian coordinates?