1.

$z = x^2 - 2y^2$

Kriittiset pisteet: $\begin{cases} \dfrac{\partial z}{\partial x} = 2x = 0 \\ \dfrac{\partial z}{\partial y} = -4y = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$

Piste $(0,0)$ kuuluu joukkoon $\{(x,y) \in \mathbb{R}^2 \mid x^2 + 2y^2 < 1\}$

ja $z = 0^2 - 2 \cdot 0^2 = 0$.

Etsitään ääriarvot joukon reunalla:

Parametrisoidaan ellipsi $x^2 + 2y^2 = 1$:

$$\begin{cases} x = \cos t \\ y = \frac{1}{\sqrt{2}} \sin t \end{cases} \qquad (0 \le t \le 2\pi)$$

Tällöin $z = \cos^2 t - 2 \cdot \frac{1}{2} \sin^2 t = \cos^2 t - \sin^2 t =$

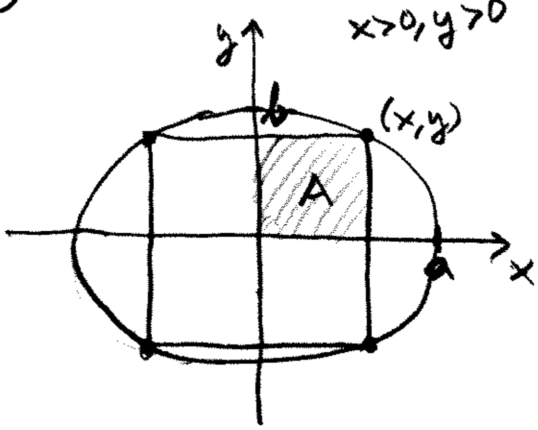
$= \cos 2t$, joten $-1 \le z \le 1$.

Arvot $z = \pm 1$ saavutetaan sopivilla $t$:n arvoilla.

($z=1$ esim, kun $t=0$, ja $z=-1$ esim, kun $t=\frac{\pi}{2}$.)

Ehdokkaat ovat siis $0, -1$ ja $1$. Näistä pienin on $-1$ ja suurin $1$.

2.




Neljäsosan pinta-ala $A = xy$

$$L(x,y,\lambda) = xy + \lambda\left(\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 - 1\right)$$

$$\begin{cases} \frac{\partial L}{\partial x} = y + \lambda \cdot \frac{2x}{a^2} = 0 & (1) \\ \frac{\partial L}{\partial y} = x + \lambda \cdot \frac{2y}{b^2} = 0 & (2) \\ \frac{\partial L}{\partial \lambda} = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 - 1 = 0 & (3) \end{cases}$$

$$(1),(2) \Rightarrow \begin{cases} xy + \frac{2\lambda}{a^2} x^2 = 0 \\ xy + \frac{2\lambda}{b^2} y^2 = 0 \end{cases}$$

$$\Rightarrow \frac{x^2}{a^2} = \frac{y^2}{b^2} \quad (*)$$

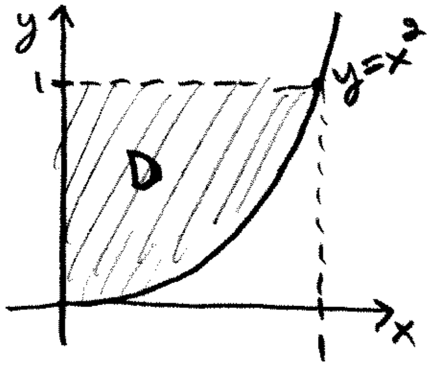
(Huom. $\lambda \neq 0$, koska tapauksessa $\lambda = 0$ olisi $x = y = 0$ eikä (3) toimi.)

$$(*) \to (3) \Rightarrow 2\left(\frac{x}{a}\right)^2 - 1 = 0 \Rightarrow x = \frac{a}{\sqrt{2}}$$

$$\Rightarrow 2\left(\frac{y}{b}\right)^2 - 1 = 0 \Rightarrow y = \frac{b}{\sqrt{2}}$$

Suurin pinta-ala on $4 \cdot \frac{a}{\sqrt{2}} \cdot \frac{b}{\sqrt{2}} = \underline{\underline{2ab}}$

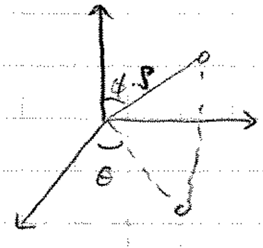
③




$$\iint_D x^3 e^{y^3}\,dA = \int_0^1\int_0^{\sqrt{y}} x^3 e^{y^3}\,dx\,dy = \int_0^1 e^{y^3} \Big/_0^{\sqrt{y}} \frac{1}{4}x^4\,dy =$$

$$= \frac{1}{4}\int_0^1 y^2 e^{y^3}\,dy = \frac{1}{4}\Big/_0^1 \frac{1}{3}e^{y^3} = \underline{\underline{\frac{1}{12}(e-1)}}$$

5:N MALLIVASTAUS




4

D: $x^2+y^2+z^2 = 4$

+1 $\Rightarrow \begin{cases} 0 \le \rho \le 2 \\ 0 \le \theta \le 2\pi \\ 0 \le \phi \le \pi \quad (1) \end{cases}$

$\left.\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases}\right\}$ +1

LISÄKSI $z > 0 \Rightarrow -\frac{\pi}{2} \le \phi \le \frac{\pi}{2} \overset{(1)}{\Rightarrow} 0 \le \phi \le \frac{\pi}{2}$

DIFFERENTIAALI: $dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$ } +1p

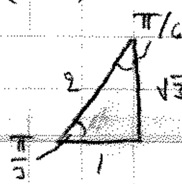
KARTION REUNAT: $3z^2 = x^2+y^2$ } +1p

$\Leftrightarrow 3\rho^2 \cos^2\phi = \rho^2 \sin^2\phi$

$\Rightarrow \tan^2\phi = 3 \Rightarrow \phi = \arctan(\pm\sqrt{3})$

$= (\pm)\arctan(\sqrt{3})$

$= \frac{\pi}{3}$ +1 } +1p




JOTEN INTEGRAALIKSI SAADAAN

$$\iiint_D z^2 \, dV = \int_0^2 \int_0^{2\pi} \int_0^{\pi/3} \rho^2 \cos^2\phi \, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$ 5p

$$= \int_0^2 \rho^4 \, d\rho \int_0^{2\pi} d\theta \int_0^{\pi/3} \sin\phi \cos^2\phi \, d\phi$$

$$= \Big/_0^2 \frac{\rho^5}{5} \cdot 2\pi \cdot \Big/_0^{\pi/3} -\frac{1}{3}\cos^3\phi$$ + 1p

$$= 2\pi \, \frac{32}{5} \left(-\frac{1}{3}\right)\left(\cos^3\frac{\pi}{3} - 1\right)$$

$$= -\frac{64\pi}{15}\Big(\underbrace{\left(\frac{1}{2}\right)^3}_{=1/8} - 1\Big) = -\frac{64\pi}{15} \cdot \left(-\frac{7}{8}\right)$$

$$= \frac{56\pi}{15}$$

6p