Appendix on set theoretical notation

When X is a set, $\mathcal{P}(X)$ denotes the family of all subsets of X (the *power set*, sometimes denoted by 2^X). The cardinality of X is denoted by $|X|$. If J is a set and $S_j \subset X$ for every $j \in J$, we write

$$\bigcup\{S_j \mid j \in J\} = \bigcup_{j \in J} S_j, \quad \bigcap\{S_j \mid j \in J\} = \bigcap_{j \in J} S_j.$$

If $f : X \to Y$, $U \subset X$, and $V \subset Y$, we define

$$f(U) := \{f(x) \mid x \in U\} \quad \text{(image)},$$

$$f^{-1}(V) := \{x \in X \mid f(x) \in V\} \quad \text{(preimage)}.$$