Institute of mathematics
Helsinki University of Technology

Arponen

Mat-1.3651 Numerical Linear Algebra (Numeerinen matriisilaskenta)

Final examination 29.08.2007

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Study programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KJO, KTA, KON, MAK, MAR, PUU, RAK, TFY, TIK, TLT, TUO, YHD.

Calculators are not allowed nor needed. Time for the exam is 3 hours. You can answer in Finnish if you wish.

1. Suppose the $m \times n$ matrix A has the form

$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix},$$

where A_1 is a square $n \times n$ invertible matrix and A_2 is $(m-n) \times n$. Let A^+ be its pseudoinverse. Prove that $\|A^+\|_2 \leq \|A_1^{-1}\|_2$.

2. Let $A \in \mathbb{C}^{m \times m}$. Show that A is unitarily similar to a diagonal matrix if and only if

$$\sum_{j=1}^{m} \sigma_j^2 = \sum_{j=1}^{m} |\lambda_j|^2,$$

where λ_j are the eigenvalues and σ_j the singular values of A.

3. Let $A \in \mathbb{C}^{m \times m}$. Show that

   (a) there exist (column) vectors $u_j, v_j \in \mathbb{C}^m$, $j = 1, \ldots, m$ such that

$$I - zA = (I - zu_m v_m^*) \cdots (I - zu_2 v_1^*)(I - zu_1 v_1^*) \quad \forall z \in \mathbb{C}.$$

   (b) The inner products $u_j^* v_j$ are the eigenvalues of A.

   Hint: Schur decomposition.

4. Suppose $A \in \mathbb{C}^{m \times m}$ is tridiagonal and Hermitian. Count the floating point operations of the QR iteration for A.