

1. A radial-basis function network calculates the function

$$F(\mathbf{x}) = \sum_{i=1}^m c_i \varphi(|\mathbf{x} - \mathbf{x}_i|).$$

Show how such a function can be realized as a multilayer perceptron network.

2. Construct a multilayer perceptron network such that when it is given the input  $\mathbf{x}$  its output is  $c\mathbf{x}$  where  $\frac{1}{c} = \sum_{j=0}^{d_0} \mathbf{x}(j)$  assuming that all components of  $\mathbf{x}$  are positive.

3. Suppose one is given a sequence of vectors  $\mathbf{x}_n$  such that  $\mathbf{x}_{n+k} = \mathbf{x}_n$  for some  $k > 1$ . Suppose the vectors  $\mathbf{y}_n$  are calculated by the formula

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \gamma(\mathbf{x}_n - \mathbf{y}_n),$$

where  $0 < \gamma < 1$ . What can one say about

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \mathbf{y}_j?$$