HUT , Institute of mathematics Gripenberg
Mat-1.196 Mathematics of neural networks
Exercise 1
15–22.1.2002

1. Why is it reasonable to call the function $\sigma(\mathbf{w}\cdot\mathbf{x}-\tau)$ a *ridge* function? Is this the case in dimension 1 as well?

2. Show that the polynomials (of one variable) of degree at most k are not dense in the spaces $\mathcal{C}([a,b])$ (continuous functions on $[a,b]$).

3. Show that if $d \geq 1$ and σ is a polynomial, then $S_d(\sigma)$ is not dense in $\mathcal{C}(\mathbb{R}^d)$ when $S_d(\sigma) = \operatorname{span}\{\, \mathbf{x}\in\mathbb{R}^d \mapsto \sigma(\mathbf{w}\cdot\mathbf{x}-\tau) \mid \mathbf{w}\in\mathbb{R}^d, \quad \tau\in\mathbb{R}\,\}$.

4. Show that

$$d(f,g) = \sum_{j=1}^{\infty} 2^{-j}\frac{\|f-g\|_{\mathcal{B}^\infty(B_j(0))}}{1+\|f-g\|_{\mathcal{B}^\infty(B_j(0))}},$$

is a metric in the space $\mathcal{C}(\mathbb{R}^d)$ where $B_j(0) = \{\,\mathbf{x}\in\mathbb{R}^d \mid |x| < j\,\}$ and $\|f\|_{\mathcal{B}^\infty(K)} = \sup_{\mathbf{x}\in K}|f(\mathbf{x})|$.